國立中正大學
113學年度碩士班招生考試
試 題

[第1節]

科目名稱	工程數學
系所組別	機械工程學系光機電整合工程

一作答注意事項一

1. (30%) The differential equation

$$\frac{dx}{dt} = x(a - bx)$$

where a and b are positive constants, has normal form $\frac{dx}{dt} = f(x)$

(a) please plot out the phase portrait (6%)

(b) please fill up the table (24%)

interval	Sign of $f(x)$	$x(t)$	arrow

2. (20%) Graph the curve traced by the vector function

$$f(a) = 2\cos a\, \boldsymbol{i} + 2\sin a\, \boldsymbol{j} + a\, \boldsymbol{k}, \ a \geq 0$$

3. (10%)

(1) Given a curve C in the $x - y$ plane, where the tangent direction at any point (x, y) is parallel to the vector $(x^2 - y^2)\, \boldsymbol{i} - 2xy\, \boldsymbol{j}$, find the equation of the curve. (5%)

(2) Find the tangent equation and normal equation for the curve $\vec{r}(t) = \frac{1+t}{t^3}\boldsymbol{i} + \frac{3+t}{2t^2}\boldsymbol{j}$ at the point $(2,2)$.

(5%)

4. (10%) Consider a simple closed curve C in the $x - y$ plane enclosing with area A , prove that

$A = -\oint_C y\,dx = \oint_C x\,dy$, and calculate the area inside $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

5. (10%) Calculate the inverse matrix of A, $A = \begin{bmatrix} 1 & -4 & 4 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}$

6. (10%) Calculate the eigenvalues and eigenvectors of matrix A, $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

7. (10%) Given that $f(z)$ is analytic at $z = a$ and $f(a) \neq 0$, and $g(z)$ at $z = a$ is a zero of order 2, determine the residue of $f(z)[g(z)]^{-1}$ at $z = a$.