國立高雄大學 113 學年度研究所碩士班招生考試試題

科目：統計學
考試時間：100 分鐘

系所：財務金融學系(財務金融組、經營管理組)
本科原始成績：100 分

是否使用計算機：是

Part I. (60%)

1. (10%) A food manufacturer uses an extruder (a machine that produces bite-size cookies and snack food) that yields revenue for the firm at a rate of $200 per hour when in operation. However, the extruder breaks down an average of two times every day it operates. If Y denotes the number of breakdowns per day, the daily revenue generated by the machine is $\mathrm{R} = 1600 - 50Y^2$. Find the expected daily revenue for the extruder. (e^{-2}=0.1353, e^{2}=7.3891)

2. (10%) The moment-generating function m(t) for a random variable Y is defined to be $\mathrm{E}(e^{tY})$. We say that a moment-generating function for Y exists if there exists a positive constant *b* such that m(t) is finite for $|t| \le b$. Let $\mathrm{m(t)} = (1/6)e^{t} + (2/6)e^{2t} + (3/6)e^{3t}$, Please find out the variance of Y, i.e. V(Y).

3. (20%) A random variable Y has the density function

$$f(y) = \begin{cases} e^{y}, y < 0 \\ 0, elsewhere \end{cases}$$

(a) Please find out $\mathrm{E}(e^{3Y/2})$.
(b) Please find out the moment-generating function for Y.
(c) Please find out $\mathrm{V}(Y)$.

4. (10%) Let (Y_1, Y_2) denote the coordinates of a point chosen at random inside a unit circle whose center is at the origin. That is, Y_1 and Y_2 have a joint density function given by

$$f(y_1, y_2) = \begin{cases} \frac{1}{\pi}, y_1^2 + y_2^2 \le 1 \\ 0, elsewhere \end{cases}$$

Please find $P(Y_1 \le Y_2)$.

5. (10%) A random sample of *n* observations, $Y_1, Y_2, \dots\dots, Y_n$, is selected from a population in which Y_i, for *i*=1, 2,……, n, possesses a uniform probability density function over the interval $(0, \theta)$ where θ is unknown. Please use the method of moments to estimate the parameter θ.

Part II. (20%)

6. (10%) For John's political poll, n=15 voters were sampled. We wish to test $H_0: p = 0.5$ against the alternative $H_a: p < 0.5$. The test statistic is Y, the number of sampled voters favoring John. Please calculate α if we select RR=$\{ y \leq 2\}$ as the rejection region. Note that $(0.5)^{15} = 0.000031$.

7. (10%) Suppose that Y represents a single observation from the probability density function given by

$$f(y|\theta) = \begin{cases} \theta y^{\theta-1}, 0 < y < 1 \\ 0, elsewhere \end{cases}$$

Please find the rejection region of the most powerful test with a significant level $\alpha = 0.05$ to test $H_0: \theta = 2$ versus $H_a: \theta = 1$.

Part III. (20%)

8. (10%) For a normal distribution with mean μ and variance σ^2=25, an experimenter wishes to test $H_0: \mu = 10$ versus $H_a: \mu = 5$. Find the sample size *n* for which the most powerful test will have $\alpha = \beta = 0.025$.

9. (10%) Consider the following model:

$$Y_i = \frac{e^{\beta_1 + \beta_2 X_i}}{1 + e^{\beta_1 + \beta_2 X_i}}$$

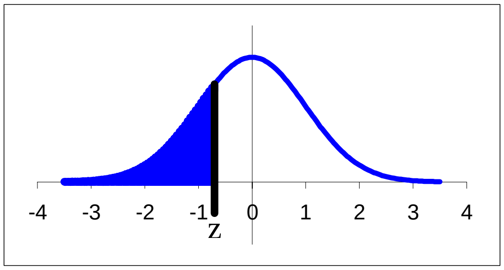
Obviously, this is not a linear regression model. Can you use any "trick" to make it a linear regression model? How would you interpret the resulting model?

Table 1a: Standard Normal Probabilities

The values in the table below are cumulative probabilities for the standard normal distribution Z (that is, the normal distribution with mean 0 and standard deviation 1). These probabilities are values of the following integral:

$$P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

Geometrically, the values represent the area to the left of z under the density curve of the standard normal distribution:



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table 2: *t*-Distribution Critical Values

The entries in the table below are the critical values $t_{n,p}$, where *n* represents the number of degrees of freedom and *p* is the upper tail probability. That is, if *T* has the *t*-distribution with *n* degrees of freedom, then the value in the table represents the number $t_{n,p}$ such that $P\left(T > t_{n,p}\right) = p$.

	Upper Tail Probability p									
d.f.	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	1.376	1.963	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
35	0.852	1.052	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
40	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
45	0.850	1.049	1.301	1.679	2.014	2.412	2.690	2.952	3.281	3.520
50	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
55	0.848	1.046	1.297	1.673	2.004	2.396	2.668	2.925	3.245	3.476
60	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
65	0.847	1.045	1.295	1.669	1.997	2.385	2.654	2.906	3.220	3.447
70	0.847	1.044	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
75	0.846	1.044	1.293	1.665	1.992	2.377	2.643	2.892	3.202	3.425
80	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
85	0.846	1.043	1.292	1.663	1.988	2.371	2.635	2.882	3.189	3.409
90	0.846	1.042	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
95	0.845	1.042	1.291	1.661	1.985	2.366	2.629	2.874	3.178	3.396
100	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
150	0.844	1.040	1.287	1.655	1.976	2.351	2.609	2.849	3.145	3.357
250	0.843	1.039	1.285	1.651	1.969	2.341	2.596	2.832	3.123	3.330
1000	0.842	1.037	1.282	1.646	1.962	2.330	2.581	2.813	3.098	3.300
∞	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291