國立中央大學113學年度碩士班考試入學試題

所別： 通訊工程學系碩士班 不分組(一般生)

科目： 通訊系統

＊本科考試禁用計算器

計算題

1. (18%) At the transmitter, during $0 \le t \le T_s$, a data bit determines the transmitted signal $x(t) = x_1(t)$ or $x_2(t)$ where $x_1(t) = -2\sin\omega_c t$ and $x_2(t) = 4\sin\omega_c t$. The received signal is $y(t) = x(t) + n(t)$ where $n(t)$ is the AWGN with double-sided PSD $N_0/2$.
(a) (4%) Find the average energy per bit E_b.
(b) (5%) Devise the coherent maximum-likelihood receiver. Also find the optimal threshold value.
(c) (5%) For (b), find the error probability in terms of the Q function and E_b/N_0.
(d) (4%) For (b), if the threshold value is zero, find the error probability in terms of the Q function and E_b/N_0.

2. (16%) Consider the transmitted signal

$$x(t) = A\cos\omega_1 t + B\cos\omega_2 t, \qquad 0 \le t \le T_s,$$

where $\cos\omega_1 t$ and $\cos\omega_2 t$ are orthogonal over the interval $[0, T_s]$. Five input data bits b_0, b_1, b_2, b_3, b_4 determine the values of A and B by $A = [2(b_0 + b_1 \times 2 + b_2 \times 4) - 7]c$ and $B = [2(b_3 + b_4 \times 2) - 3]c$ where c is a constant. The received signal is $x(t) + n(t)$ where $n(t)$ is the AWGN with double-sided PSD $N_0/2$. Assume that the coherent maximum-likelihood detector is used.
(a) (4%) Find the average energy per symbol E_s.
(b) (6%) What is the symbol error probability in terms of the Q function and E_s/N_0?
(c) (6%) What is the approximate bit error probability in terms of the Q function and E_b/N_0?

3. (10%) Consider the discrete memoryless channel with three inputs x_0, x_1, x_2 and three outputs y_0, y_1, y_2. The channel transition probabilities are $p(y_j|x_i) = \begin{cases} p & \text{if } j = i \\ 1-p & \text{if } j = (i+1) \mod 3 \\ 0 & \text{if } j = (i+2) \mod 3 \end{cases}$ for $i, j \in \{0, 1, 2\}$ where $0 \le p \le 1$ is a constant and "mod" is the modulo operation.
(a) (4%) Find the channel capacity C.
(b) (3%) Determine the value of p to maximize C. What is the maximum value of C?
(c) (3%) Determine the value of p to minimize C. What is the minimum value of C?

4. (6%) At the transmitter, three signaling intervals transmit one data bit b. During $0 \le t \le 3T_s$, the transmitted signal $x(t)$ is $A\cos\omega t$ if $b = 1$ or 0 if $b = 0$. The received signal is $x(t) + n(t)$ where $n(t)$ is the AWGN with double-sided PSD $N_0/2$. What is the bit error probability of the coherent optimal receiver in terms of the Q function? Please explain your reason.

注意：背面有試題

國立中央大學113學年度碩士班考試入學試題

所別： 通訊工程學系碩士班 不分組(一般生)

科目： 通訊系統

＊本科考試禁用計算器

5. (25%) In a DSP quadrature modulator system as shown below with the DAC outputs $s_{B,\delta}(t) = s_{I,\delta}(t) + j \cdot s_{Q,\delta}(t) = \sum_{m=-\infty}^{\infty}(s_I[m] + j \cdot s_Q[m]) \cdot \delta(t - m \cdot T_S)$, and the lowpass filter (LPF) having a frequency response $H_{LP}(f) = \Im\{h_{LP}(t)\} = A \cdot \begin{cases} 1, & |f| \le f_{pass} \\ 1 - \frac{|f| - f_{pass}}{f_{stop} - f_{pass}}, & f_{pass} < |f| \le f_{stop} \\ 0, & f_{stop} < |f| \end{cases}$,

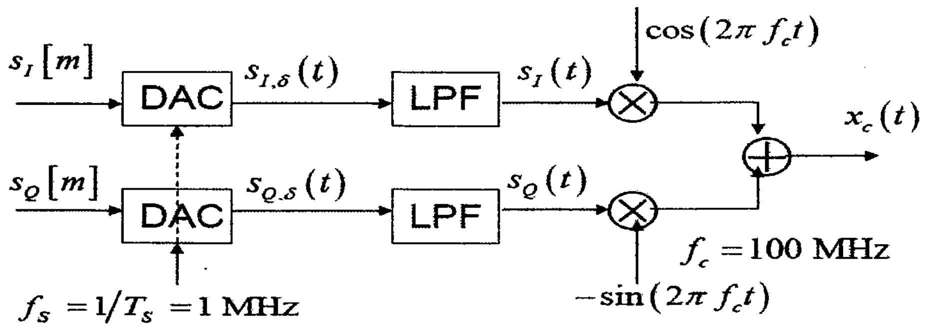
{Hint (Sampling Theorem): $s_B[m] = \hat{s}_B(m \cdot T_S)$, $S_{B,\delta}(f) = \Im\{s_{B,\delta}(t)\} = f_S \cdot \sum_{k=-\infty}^{\infty} \hat{S}_B(f - k \cdot f_S)$, $\hat{S}_B(f) = \Im\{\hat{s}_B(t)\}$, $f_S = \frac{1}{T_S}$}, $\Im\{\ \}$: *denotes the Fourier Transform*

(a) plot $|\hat{S}_{B,\delta}(f)|^2$ in the range $|f| < 2 \cdot f_S$ when $|\hat{S}_B(f)|^2 = \begin{cases} \frac{f+f_0}{f_0}, & |f| \le f_0 \\ 0, & f_0 < |f| \end{cases}$ and $f_S = 4 \cdot f_0$ (7%)

(b) find the frequency response specification, i.e., $\{f_{pass}, f_{stop}, A\}$ of the LPF with **minimum f_{pass} and maximum f_{stop}** such that $s_B(t) = s_I(t) + j \cdot s_Q(t) = \hat{s}_B(t)$ for the signal given in (a). (6%)

(c) find the **formula of $X_c(f) = \Im\{x_c(t)\}$ in terms of $S_B(f) = \Im\{s_B(t)\}$** (Hint: $x_c(t) = \text{real}\{s_B(t) \cdot exp(j \cdot 2\pi \cdot f_c \cdot t)\}$, $s_B(t) = s_I(t) + j \cdot s_Q(t)$, $\text{real}\{x\} = \frac{1}{2}(x + x^*)$, $j = \sqrt{-1}$) (6%);

(d) find the **formula of $s_B(t)$ in terms of the message signal $\mathrm{m}(t)$** when $x_c(t)$ is an FM signal with an instantaneous frequency deviation $f_D = 2\pi \cdot f_0 \cdot \mathrm{m}(t)$. (6%)



DAC: digital-to-analog converter
LPF : Lowpass Filter

注意：背面有試題

＊本科考試禁用計算器

6. (25%) In a DSP quadrature demodulator system as shown below with the LPF having a frequency response $H_{LP}(f) = \Im\{h_{LP}(t)\} = \begin{cases} 2, & |f| < 0.5 \cdot f_S \\ 0, & \text{otherwise} \end{cases}$,

(Hint: $r_B(t) = r_I(t) + j \cdot r_Q(t) = \{r_c(t) \cdot exp(-j2\pi \cdot (f_c - 0.01 \cdot f_S) \cdot t)\} * h_{LP}(t)$)

(a) **find the formula of $r_B(t)$ in terms of $\hat{r}_B(t)$** when $r_c(t) = \text{real}\{\hat{r}_B(t) \cdot exp(j \cdot 2\pi \cdot f_c \cdot t)\}$ and $\hat{r}_B(t)$ has a lowpass bandwidth less than $0.25 \cdot f_S$. (6%)

(b) **find the formula of $r_I[m]$ and $r_Q[m]$** when $r_c(t) = \cos(2\pi \cdot f_m \cdot t) \cdot cos(2\pi \cdot f_c \cdot t + \theta)$ and $f_m = 100\ KHz$ (7%). (Hint: find **$\hat{r}_B(t)$ first)**

(c) **find the formula of $r_I[m]$ and $r_Q[m]$** when $r_c(t) = 2 \cdot cos(2\pi \cdot (f_c + f_m) \cdot t)$ and $f_m = 100\ KHz$ (6%) (Hint: find **$\hat{r}_B(t)$ first)**

(d) **find the formula of $R_M(f) = \Im\{\{r_c(t) \cdot exp(-j2\pi \cdot (f_c - 0.01 \cdot f_S) \cdot t)\}\}$ in terms of $\hat{R}_B(f) = \Im\{\hat{r}_B(t)\}$ when** $r_c(t) = \text{real}\{\hat{r}_B(t) \cdot exp(j \cdot 2\pi \cdot f_c \cdot t)\}$ (6%)