國立臺灣大學 113 學年度碩士班招生考試試題

1. (20 points) Solve the following initial value problem:

$$\begin{cases} u_t(x,t) + 2u_x(x,t) = 2x - t & \text{in } \mathbb{R} \times (0,\infty), \\ u(x,0) = 10x. \end{cases}$$

2. (20 points) Suppose $u(x)$ is a harmonic function (i.e., $\Delta u(x) = 0$) defined on the open set $\Omega \subset R^3$ (i.e., $\Delta u(x) = 0$) . Suppose $B(x_0, r) \subset \Omega$, where $B(x_0, r)$ is the open ball with radius $r > 0$ centered at $x_0 \in \mathbb{R}^3$. Show that

$$|\frac{\partial u}{\partial x_1}(x_0)| \le \frac{3}{r} \sup_{\partial B(x_0,r)} |u|.$$

Here we use the notation $x = (x_1, x_2, x_3)$. Hint : The mean value property and the divergence theorem.

3. (20 points) Set

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{pmatrix},\ X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix},\ f(t) = \begin{pmatrix} 3t \\ 0 \\ 2e^t \end{pmatrix}.$$

Find the solution to the differential system

$$X'(t) = AX(t) + f(t)$$

with the initial condition $x_1(0) = 1$, $x_2(0) = 0$ and $x_3(0) = 2$.

4. (40 points) Let $\delta(t)$ be the Dirac delta function. Solve the following differential equations :

(a)

$$y''' + 6y'' + 10y' = 2\delta(t-5)$$

with initial condition $y(0) = y'(0) = y''(0) = 0$.

(b)

$$x^{(3)}(t) - x''(t) - 2x'(t) = \cos(t)$$

with the initial condition $x(0) = x'(0) = x''(0) = 0$.

試題隨卷繳回