

國立臺灣師範大學 112 學年度碩士班招生考試試題

科目：工程數學

適用系所：機電工程學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

【試題 1】 (15 分)

$$y'' + y' - 2y = e^x$$

(a) Find a general solution.

(b) Is this system stable or unstable? Please give an explanation.

【試題 2】 (15 分)

$$A = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}, \det A = 5, \text{ find (a) } \det(-4A), \text{ (b) } \det(A^{-1}), \text{ (c) } \det(A^2),$$

$$\text{(d) } \det((3A^{-1})^T), \text{ (e) } \det \begin{bmatrix} t & r & s \\ w & u & v \\ z & x & y \end{bmatrix}$$

【試題 3】 (15 分)

Solve the following initial value problem by the Laplace transform.

$$y'' + 3y' + 2y = u(t-1) + \delta(t-2), \quad y(0) = 0, \quad y'(0) = 1$$

【試題 4】 (20 分)

Use convolution theorem to solve the following equations:

$$\text{(a) } f(t) = \cos t + \int_0^t f(\tau) e^{-2(t-\tau)} d\tau, \text{ find } f(t) = ?$$

$$\text{(b) } H(s) = \frac{2}{(s+1)(s+2)(s+4)}, \text{ find } L^{-1}[H(s)] = h(t) = ?$$

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【試題 5】 (20 分)

(a) Let $\vec{F} = (x^2, y^2, xy^2)$ be a force, calculate the work (ie, $W = \int_C \vec{F} \cdot d\vec{r}$) done in the displacement along the helix $C: \vec{r} = [2\cos t, 2\sin t, 6t]$ from $A(2, 0, 0)$ to $B(0, 2, 3\pi)$.

(b) What is $\int_C (xy + z^2) ds$, where C is the arc of the helix $x = \cos t, y = \sin t, z = t$, Which joints the points $(1, 0, 0)$ and $(-1, 0, \pi)$?

【Hint】 $ds = |d\vec{r}|, \sin \theta \cos \theta = \frac{\sin 2\theta}{2}$

【試題 6】 (15 分)

(a) Find the Fourier series of $f(x) = |x|, -\pi < x < \pi$

(b) $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = ?$