

# 國立臺灣師範大學 112 學年度碩士班招生考試試題

科目：機率與統計

適用系所：數學系

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

- (16 分) If a sample of two balls are to be selected from an urn contains 10 balls numbered from 1 to 10. Let  $X$  and  $Y$  be the numbers of the first and second selected balls respectively.
  - Give  $P(X \geq Y)$  under sampling with replacement case.
  - Give  $P(X \geq Y)$  under sampling without replacement case.
- (24 分) Let  $X_1, \dots, X_n$  be a random sample from a  $\Gamma(1, \theta)$  distribution with mean  $\theta$ .
  - Give the maximum likelihood estimator of  $\theta$ .
  - Give an approximate  $(1-\alpha)100\%$  confidence interval of  $\theta$ .
  - Give an exact  $(1-\alpha)100\%$  confidence interval of  $\theta$ .
- (16 分) Two cards are randomly chosen without replacement from an ordinary deck of 52 cards. Let  $X$  denote the number of spades chosen.
  - Give  $E(X | \text{the ace of spades is chosen})$ .
  - Give  $E(X | \text{at least one spade is chosen})$ .
- (24 分) Let  $X_1, \dots, X_n$  be a random sample from a Poisson( $\theta$ ) distribution with mean  $\theta$  and  $Y = g(\bar{X})$ , where  $g(\cdot)$  is some differentiable function and  $\bar{X}$  is the sample mean. By using the first order Taylor approximation,  $Y$  can be written as
$$Y = g(\bar{X}) \approx g(\theta) + g'(\theta)(\bar{X} - \theta).$$
  - Give  $E(Y)$  under this approximation.
  - Give  $Var(Y)$  under this approximation.
  - Give  $g(\cdot)$  if  $Var(Y) \approx 1$ .
- (20 分) If a random variable  $X$  is either from  $U(0, 2)$  or  $U(1, 3)$  distribution, we need to test the hypothesis of  $H_0: X \sim U(0, 2)$  versus  $H_a: X \sim U(1, 3)$ . Please derive an optimal test by using the Neyman-Pearson Theorem. You must specify the testing rule, including test statistic and critical region for significance level  $\alpha$ .

(試題結束)