

# 國立臺北科技大學 112 學年度碩士班招生考試

系所組別：1301、1302、1303 車輛工程系碩士班

## 第一節 工程數學 試題

第 1 頁 共 1 頁

### 注意事項：

1. 本試題共 5 題，每題 20 分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Please find a general solution for the following differential equation. Show the steps of derivation. (20%)

$$xy' = x + y \quad (\text{hint: Set } y/x = u)$$

2. Given the following differential equation

$$y''' - 2y'' - y' + 2y = 0$$

If one basis  $y_1 = e^{-x}$  is already known, please find the other basis of solutions. (20%)

3. For a linear and homogeneous system in terms of matrix form (20%)

$$\mathbf{y}' = \mathbf{A}\mathbf{y} = \begin{bmatrix} 0 & 1 \\ -0.75 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Please find its general solution and show the details of derivation.

4. Please using the Laplace transform and showing the details for solving the following differential equation: (20%)

$$(\text{Hint: } \mathcal{L}\{\sin \omega t\} = \frac{\omega}{s^2 + \omega^2}, \quad \mathcal{L}\{\cos \omega t\} = \frac{s}{s^2 + \omega^2})$$

$$y'' + 9y = 10e^{-t}, \quad y(0) = y'(0) = 0$$

5. Given a matrix as follows: (20%)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$$

- (a) Find the rank of matrix A. (5%)
- (b) Find a basis for the row space and the column space, respectively. (5%)
- (c) Calculate the determinant. (5%)
- (d) Does this matrix have the inverse matrix? Show your reasons. (5%)