

國立中正大學

112 學年度碩士班招生考試

試題

[第 2 節]

科目名稱	機率
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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系所組別：通訊工程學系-通訊甲組

- 1) (10 points) Alice has three children. Assume that all eight possible arrangements of boy “b” and girl “g” in the order of birth, $\{ggg, bgg, gbg, ggb, bbg, bgb, gbb, bbb\}$, are equally probable.
- (5 points) What is the probability that Alice has two girls and one boy?
 - (5 points) Given the information that at least one of Alice’s children is a boy and the younger child is not a girl, what is the probability that Alice has two girls and one boy?

- 2) (15 points) Let X be the number of heads in 10 tosses of a fair coin.

- (5 points) Find the probability mass function of X .
- (5 points) Find the mean of X .
- (5 points) Find the variance of X .

- 3) (20 points) A random variable X has the following probability density function (pdf):

$$f_X(x) = \begin{cases} c(1 - x^4), & \text{for } -1 \leq x \leq 1, \\ 0, & \text{elsewhere,} \end{cases}$$

where c is a constant.

- (5 points) Find c .
 - (5 points) Find the probability that $X > 0$.
 - (5 points) Find the cumulative distribution function (cdf) $F_X(x)$ of X .
 - (5 points) Find the mean of X .
- 4) (15 points) Consider the following joint pdf of two random variables X and Y :
- $$f_{X,Y}(x, y) = \begin{cases} x + y, & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$
- (5 points) Find the marginal pdf of X .
 - (5 points) Find the probability that “ $Y \geq X + 0.5$ ”.
 - (5 points) Are X and Y dependent or independent? Please explain your answer. (0 point if the explanation is incorrect.)
- 5) (20 points) Let X be a normal (Gaussian) random variable with mean 40 and variance 16. Consider $Y = aX + b$, where a and b are two constants to be designed such that Y has zero mean and unit variance.

- (10 points) Find a and b .
- (5 points) Find the pdf of Y .
- (5 points) Find the characteristic function of Y .

- 6) (20 points) Let X be a continuous random variable with cdf $F_X(x)$ and pdf $f_X(x)$. Consider $Y = X^2$.

- (5 points) The event $\{Y \leq y\}$ is equivalent to what event involving X itself?
- (5 points) Use part a) to find the cdf of Y .
- (5 points) Use part b) to find the pdf of Y .
- (5 points) If X denotes the amplitude of a radio signal with the following pdf:

$$f_X(x) = \frac{x}{\alpha^2} e^{-x^2/2\alpha^2}, \quad x > 0, \quad \alpha > 0,$$

where α is a constant, use part c) to find the pdf of the squared envelope Y .