國立中正大學
112學年度碩士班招生考試
試題
[第2節]

科目名稱	線性代數
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

國立中正大學 112 學年度碩士班招生考試試題

科目名稱：線性代數　　　　　　　　　　　　　　　　　　本科目共 1 頁　第 1 頁

系所組別：通訊工程學系-通訊甲組

Let $\mathbf{A}=[\mathbf{A}^{(1)}\ \mathbf{A}^{(2)}\ \mathbf{A}^{(3)}]=\begin{bmatrix}1&0&0\\0&2&0\\1&0&2\end{bmatrix}$, where $\mathbf{A}^{(1)}$, $\mathbf{A}^{(2)}$ and $\mathbf{A}^{(3)}$ are ordered column vectors of **A**,

$\mathbf{B}=[\mathbf{B}^{(1)}\ \mathbf{B}^{(2)}\ \mathbf{B}^{(3)}]=\begin{bmatrix}2&0&2\\0&1&0\\0&0&1\end{bmatrix}$, where $\mathbf{B}^{(1)}$, $\mathbf{B}^{(2)}$ and $\mathbf{B}^{(3)}$ are ordered column vectors of **B**, and

$\mathbf{I}_3=[\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3]=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$, where $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ are ordered column vectors of $\mathbf{I}_3$.

1. Show your answers with details
   a. (5 pts.) The sum of all eigenvalues in **A**.
   b. (5 pts.) The geometry multiplicities of **A**.
   c. (5 pts.) The product of all eigenvalues in **B**.
   d. (5 pts.) The inverse matrix of **B** with the augmented matrix $[\mathbf{I}_3|\mathbf{B}]$ and Gauss-Jordan elimination.
   e. (15 pts.) The solution of $\mathbf{A}[x_1\ x_2\ x_3]^T=[1\ 2\ 3]^T$ with Cramer's rule.

2. In $\boldsymbol{R}^3$, find the results with details.
   a. (5 pts.) The transition matrix from the standard basis $\underline{e}=\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ to the basis $\underline{A}=\{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}\}$.
   b. (5 pts.) The coordinate vector with the basis $\underline{A}=\{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}\}$ corresponding to $(1\ 2\ 3)_{\underline{e}}$ with the standard basis $\underline{e}=\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$.
   c. (10 pts.) The coordinate vector with the basis $\underline{A}=\{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}\}$ corresponding to $(1\ 2\ 3)_{\underline{B}}$ with the basis $\underline{B}=\{\mathbf{B}^{(1)}, \mathbf{B}^{(2)}, \mathbf{B}^{(3)}\}$.
   d. (10 pts.) The transition matrix from the standard basis $\underline{B}=\{\mathbf{B}^{(1)}, \mathbf{B}^{(2)}, \mathbf{B}^{(3)}\}$ to $\underline{A}=\{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}\}$.

3. The inner product is $\langle\mathbf{U},\mathbf{V}\rangle=\mathrm{tr}(\mathbf{U}^T\mathbf{V})$ where **U** and **V** are in the real vector space $\mathbf{M}_{3\times 3}$, and tr(**X**) is the trace of the matrix **X**.
   a. (5 pts.) Find the inner product of the identity matrix ($\mathbf{I}_3$) and **A**.
   b. (10 pts.) Prove or disprove that the additivity axiom holds with this inner product.
   c. (10 pts.) Find the norm-2 length of **B**.
   d. (10 pts.) Show the cosine of the angle between the matrices **A** and **B** with details.