國立中正大學
112學年度碩士班招生考試
試 題

[第2節]

科目名稱	機率與統計
系所組別	數學系統計科學

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

國立中正大學 112 學年度碩士班招生考試試題

科目名稱：機率與統計　　本科目共 1 頁　第 1 頁

系所組別：數學系統計科學

1. (10%) X has mean μ and variance σ^2. If a and b are constants and $Y=aX+b$, find its mean and variance.

2. (10%) If $E(X^n)=0.8, \forall n \in \mathbb{N}$, find $P(X<1)$.

3. (10%) Let X denote that tangent of an angle chosen at random from $(-\frac{\pi}{2},\frac{\pi}{2})$, what is the p.d.f. of X?

4. (10%) Let $X|P=p \sim Bin(n,p)$ and $P \sim Beta(\alpha,\beta)$, what is the conditional distribution of $P|X=x$?

5. (10%) Suppose that X and Y are independent gamma random variables with $E(X)=\frac{\alpha}{\lambda}$ and $E(Y)=\frac{\beta}{\lambda}$. What is the distribution of $\frac{Y}{X+Y}$?

6. (10%) Let $X_1,...,X_n$ be a random sample from a distribution with the p.d.f. $f(x;\theta)=\frac{3\theta^3}{x^4}, x \geq \theta$. Find a one-dimensional sufficient statistic for θ.

7. (10%) Let (X_i,Y_i) be independent pairs, $i=1,2,...,n$, with X_i and Y_i are independent exponential random variables with $E(X_i)=\frac{1}{\theta\lambda}$ and $E(Y_i)=\frac{1}{\lambda}$. Find the maximum likelihood estimators for θ and λ.

8. (10%) Let $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$ be the sample mean and variance of a random sample from a Poisson distribution with mean λ. What is $E(S^2|\bar{X})$?

9. (10%) Let $X_1,...,X_n$ be a random sample from a normal distribution $N(\mu,\sigma^2)$, where μ is known. Find a level $(1-\alpha)$ confidence interval for σ.

10. (10%) Refer to the previous question, find a uniformly most powerful test with significance level α for testing $H_0:\sigma \leq \sigma_0$ vs. $H_1:\sigma > \sigma_0$.