國立雲林科技大學 112 學年度 碩士班招生考試試題

系所：電子系

科目：工程數學(2)

本試題共 7 題，每題得分如各題中所示，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. Please solve for $y = y(x)$.

   (a) (05%) $xy' + 3y = 2x$

   (b) (05%) $y'' + 5y' + 6y = 0$

   (c) (05%) $x^2y'' + 1.5xy' - 0.5y = 0$

2. (15%) Find the integration factor and solution of the ODE equation

$$(3x^2y + 6xy + \frac{y^2}{2})dx + (3x^2 + y)dy = 0$$

3. (10%) Solve ODE solution of $y'' + 2y' + y = xe^{-x}$

4. (10%) Laplace equation:

   (a) If $f(t) = (t+2)^2$, $t \geqq 0$, please find $L[f(t)]$

   (b) $F(S) = \frac{3}{(S+3)} + \frac{3S}{S^2+5}$, please find $L^{-1}[F(S)]$ 。

5. A transform $T: \mathcal{R}^4 \to \mathcal{R}$ is defined as follows,

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

   (a) (10%) Prove that T is a linear transformation.

   (b) (05%) Find the transformation matrix $\mathbf{A}$ to make $T(\mathbf{x}) = \mathbf{Ax}$.

   (c) (05%) Find the kernel of $\mathbf{A}$.

6. (10%) Let $\mathbf{A} = \begin{bmatrix} 1 & -6 & 4 \\ -3 & 8 & -2 \\ 4 & 7 & h \end{bmatrix}$, find the value of h to make $\mathbf{A}$ invertible.

7. Given $\mathbf{A} = \begin{bmatrix} 0.4 & -0.3 \\ 0.4 & 1.2 \end{bmatrix}$,

   (a) (10%) Find an invertible matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ to make $\mathbf{A} = \mathbf{PDP^{-1}}$.

   (b) (05%) Find $\lim_{n\to\infty} \mathbf{A}^n$.

   (c) (05%) Find the eigenvalues of $\mathbf{A^{-1}}$