

科目：機率與統計

系所組：數學系乙組

1. Let X and Y be two random variables such that $Y=1000X+98765$. Find the correlation coefficient of X and Y , and explain the reason for your answer. (10%)
2. Let $f(x)=\begin{cases} c/\{x\ln(x)[\ln(\ln(x))]\} & \text{if } e^e < x < 100 \\ 0 & \text{else} \end{cases}$ be a probability density function of a random variable X . Find the value of c . (10%)
3. Let Y be the number of flaws on the surface of a randomly selected product. Assume Y has a Poisson distribution with mean 5, find the probability that Y is less than 2. (10%)
4. (a) Define a memoryless property of a distribution. (10%)
(b) Is there any continuous type or discrete type of distribution which has memoryless property? Name one for each type if possible. (10%)
5. Let X_1, X_2, \dots, X_n be a random sample from $U[\theta - 1, \theta + 1]$, find the MLE of θ . (10%)
6. Denote \bar{X} and S_1 as the sample mean and sample standard deviation of the random sample X_1, X_2, \dots, X_m , respectively. Define \bar{Y} and S_2 similarly from random sample Y_1, Y_2, \dots, Y_n . Assume $E(X_i) = \mu_1, i = 1, \dots, m$ and $E(Y_j) = \mu_2, j = 1, \dots, n$. Test $H_0: \mu_1 - \mu_2 = 0$ versus $H_1: \mu_1 - \mu_2 > 0$ at level approximate 0.05 using the following data from the two random samples: $m=60, \bar{x} = 42500, s_1 = 2200, n=60, \bar{y} = 40400, s_2 = 1900$. (10%)
7. (a) Define p-value of a testing problem. (10%)
(b) How do you use p-value in a testing problem when the significance level is set at 0.05? (10%)
8. If $(\bar{X} - 1.96S/10, \bar{X} + 1.96S/10)$ is an approximate confidence interval of the mean of a distribution based on a random sample X_1, X_2, \dots, X_{100} , where \bar{X} and S are the sample mean and sample standard deviation, respectively. What can you say about the confidence level of such problem. (10%)

Remark:

$$z_{0.1} = 1.285, z_{0.05} = 1.645, z_{0.025} = 1.96;$$

$$t_{0.1; 60} = 1.25, t_{0.05; 60} = 1.65, t_{0.025; 60} = 2$$

※ 注意：1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部份可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。