

科目：微積分

系所組：數學系

1. (20 points) Evaluate the limits (a) $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$ (b) $\lim_{x \rightarrow \infty} \frac{\sin(x)}{x}$
 (c) $\lim_{x \rightarrow 0^+} x \sin \frac{1}{x}$ (d) $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$

2. (15 points) Show that $\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$, where $x \in (-1, 1)$.

3. (15 points) Calculate the derivative $\frac{d}{dx} \int_x^x [t \sin(xt) + e^{t^2}] dt$.

4. (15 points) Find the absolute maximum and minimum values of

$$f(x, y) = 2 + 2x + 6y - x^2 - 9y^2$$

on the triangular region in the first quadrant bounded by the lines $x = 0, y = 0, y = 3 - x/3$

5. (15 points) Find $\frac{\partial u}{\partial y}$ at the point $(x, y, z) = (-1, 2, 1)$ if $u = x^2 + y^2 + z^2, z^3 - xy + xz + x^3 = 1$, and x and y are the independent variables.

6. (10 points) Evaluate the integral $\int x^4 \sqrt{x^3 + 1} + 2x \sin^{-1}(x^2) dx$

7. (10 points) Evaluate the line integral $\oint_C xy dy - y^2 dy$, where C is the square cut from the first quadrant by the lines $x = 2$ and $y = 2$

※ 注意：1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部份可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。