

科目： 工程數學

系所組：物理系碩士班光電組

1. Solve the ODE: $(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0$. (20%)

2. Solve the ODE: $y'' + 4y' + 4y = e^{-x} \cos x$. (20%)

3. Find the Laplace transform for the following function $f(t)$:

(a) $f(t) = e^{3t} \cos(\pi t)$ (7%) (b) $f(t) = \begin{cases} \cosh 2t, & \text{for } 0 < t < 2 \\ 0, & \text{otherwise} \end{cases}$ (7%) (c) $f(t) = te^{-2t} \sin t$ (6%)

4.(a) Show that $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\pi/a}$. (5%)

(b) Find the Fourier transform of Gaussian function $f(x) = e^{-x^2}$. (15%)

5. (a) Calculate the integral $\oint_C \frac{e^z}{(z-1)^2 (z+4)^2} dz = ?$ where the contour $C: |z|=1.1$.

(z is a complex number and the contour is counterclockwise.) (10%)

(b) Find the Laurent series of $f(z) = \frac{1}{z^3 - z^4}$ for the region of $|z| > 2$. (10%)

※ 注意：1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部份可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。