科目： 近代物理 系所組：物理系物理組

1. (10)

An electron has a kinetic energy of $E = 1$ eV. Calculate the de Broglie wavelength?

Hint: $h = 6.626\times10^{-34} J\cdot s$; $m_e = 0.911\times10^{-30} kg$; $1eV = 1.602\times10^{-19} J$

2. (10)

An electronic state has a life time of 2.6×10^{-6} s. What is the uncertainty in the energy of that electronic state?

Hint: $\hbar = 1.06\times10^{-34} J\cdot s$

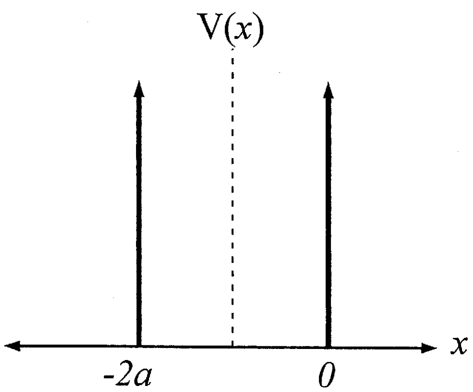
3. (10)

A laser beam of wavelength λ=35.3 nm illuminates on an insulator surface and some electrons were emitted with kinetic energy of E=1.7 eV. Calculate the work function (ϕ) of the insulator in eV.

4. (10)

An electron of mass m with total energy $\boldsymbol{E}$ was contained in closed box, which can be considered as an infinite one-dimensional potential well;

$$V(x) = \begin{cases} 0 & 0 > x > -2a \\ \infty & x = -2a,\ 0 \end{cases}$$



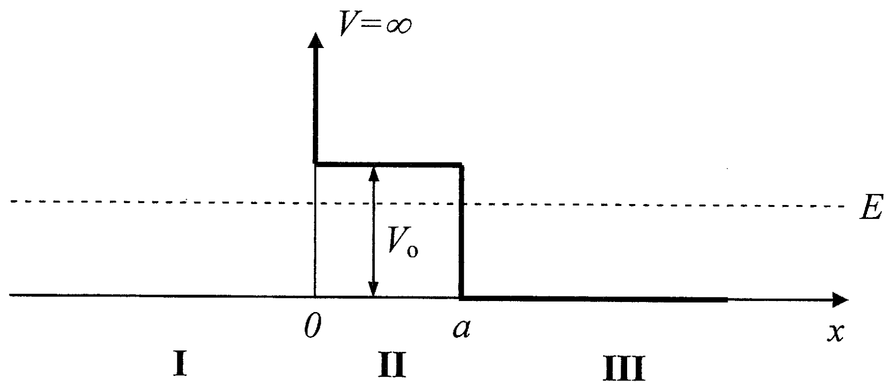
Find the allowed energies E_n of electron inside the box:

Hint: $\int_0^{n\pi} \sin^2(y)dy = \frac{n\pi}{2}$; $\int_{\vartheta_1}^{\theta_2} y\sin^2(y)dy = [\frac{y^2}{4} - \frac{y\sin(2y)}{4} - \cos(2y)]_{y=\theta_1}^{y=\theta_2}$

5. (10)

Consider a potential barrier as shown in Figure and a particle has an energy E in the range of $0 < E < V_0$. Find wave functions of the Schrödinger equation in the various regions (I, II, and III);

Hint: $[\frac{d^2}{dx^2}+\frac{2m}{\hbar^2}(E-V_o)]u(x)=0$, $k^2=2mE/\hbar^2$, $q^2=\frac{2m}{\hbar^2}(V_o-E)$



6. (10)

Follow above problem and use the boundary conditions at x=0 and x=a. Find the correct matching solutions;

7. (10)

Consider a spherical harmonic function $Y_{l,m}(\theta,\phi)=A\sin\theta\cos\theta e^{i\phi}$, which is an eigenfunction of L^2 and L_z. L and L_z are orbital angular momentum and its z-component. Find the corresponding quantum numbers of l and m;

Hint: $L^2=-\hbar^2(\frac{\partial^2}{\partial\theta^2}+\cot\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2})$; $L_z=-i\hbar\frac{\partial}{\partial\phi}$

8. (10)

Find the average distance $\langle r \rangle$ of the electron from the hydrogen nucleus for the ground state (n=1) with the radial eigenfunction, $R_{10} = 2(\frac{1}{a_o})^{3/2} e^{-r/a_o}$.

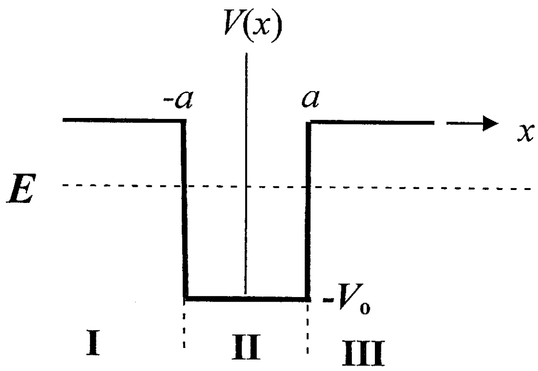
Hint: (1) Partial integral: $f = r^3$, $dg = e^{-2r/a_o} dr$, $df = 3r^2 dr$, $g = \frac{-a_o}{2} e^{-2r/a_o}$

(2) $\int_0^\infty r^2 e^{-2r/a_o} dr = [\frac{-a_o}{2} e^{-2r/a_o} (r^2 - \frac{-2a_o}{2} r + \frac{a_o^2}{2})]_0^\infty$

9. (20)

Consider the bound states of a particle, which is trapped in a potential. The total energy of particle is $E < 0$. Find the solutions of the bound states.

$$V(x) = \begin{cases} 0 & for \quad x < -a \\ -V_o & for \quad -a < x < a \\ 0 & for \quad x > a \end{cases}$$



Hint: $-\frac{\hbar^2}{2m}\frac{d^2u(x)}{dx^2} + V(x)u(x) = Eu(x)$, $\kappa^2 = \frac{2m}{\hbar^2}|E|$, $q^2 = \frac{2mV_o}{\hbar^2} - \kappa^2$

※ 注意：1.考生須在「彌封答案卷」上作答。
2.本試題紙空白部份可當稿紙使用。
3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。