元智大學 103 學年度研究所 **碩士班** 招生試題卷

系(所)別：	管理學院財務金融暨會計碩士班	組別：	財務金融碩士學程	科目：	微積分	用紙第 1 頁共 1 頁

●可使用現行『國家考試電子計算器規格標準』規定第一類之計算機

1. (32%) Computing the following:

   (a) (8%) $\lim_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+5}$

   (b) (8%) $\frac{d}{dx}\left[(1+x)^{\sqrt{1+x}}\right]$

   (c) (8%) $\int(\ln x)^2\,dx$

   (d) (8%) $\int_0^3\int_{\frac{y}{3}}^1 ye^{x^3}\,dx\,dy$

2. (10%) Prove $\int f^{-1}(x)\,dx = xf^{-1}(x)-F\left(f^{-1}(x)\right)$, where $F'(x)=f(x)$.

3. (24%) Assume x is univariate normally distributed and the definitions of the corresponding probability density function as $f(x)=\left(1/\sqrt{2\pi\sigma^2}\right)e^{-(x-\mu)^2/2\sigma^2}$.

   (a) (8%) Find the extrema of $f(x)$.

   (b) (8%) Find the points of inflection of $f(x)$.

   (c) (8%) Calculate $\int_{-\infty}^{\infty}e^{tx}f(x)\,dx$.

4. (24%) The Black-Scholes formula of a call option is defined as follows:

   $C=SN(d_1)-Ke^{-rT}N(d_2)$,

   where $d_1=\frac{\ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$, $d_2=d_1-\sigma\sqrt{T}=\frac{\ln\left(\frac{S}{K}\right)+\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$, $N(.)$ is the cumulative distribution function of the standard normal distribution defined as $N(d)=\int_{-\infty}^{d}\left(1/\sqrt{2\pi}\right)e^{-x^2/2}dx$, and (S,r,K,T,σ) are input parameters and are given. Please find (a) $\frac{\partial C}{\partial S}$, (b) $\frac{\partial C}{\partial \sigma}$, and (c) $\frac{\partial C}{\partial r}$.

5. (10%) Define a function of y as follows:

   $$f(y)=\sum_{i=1}^{2}p_ie^{a_i(1-y)+\frac{b_i(1-y)^2}{2}}-\sum_{i=1}^{2}p_ie^{-a_iy+\frac{b_iy^2}{2}},$$

   where $-\infty<a_i<\infty, 0<b_i<\infty, \sum_{i=1}^{2}p_i=1$, and $(a_1/b_1)\neq(a_2/b_2)$. Please prove the function $f(y)$ has a unique solution.

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