國立清華大學 101 學年度碩士班考試入學試題

系所班組別：動力機械工程學系碩士班 乙組(電控組)

考試科目（代碼）：控制系統 (1202)

Q1 The transfer function in Figure 1 is described as:

$$G(s)=\frac{a}{s+b}e^{-st}$$

(a) What is the break frequency ω_b rad/s? (2%)

(b) What is the value of a? (3%)

(c) What is the value of b? (2%)

(d) What is the value of t? (3%)

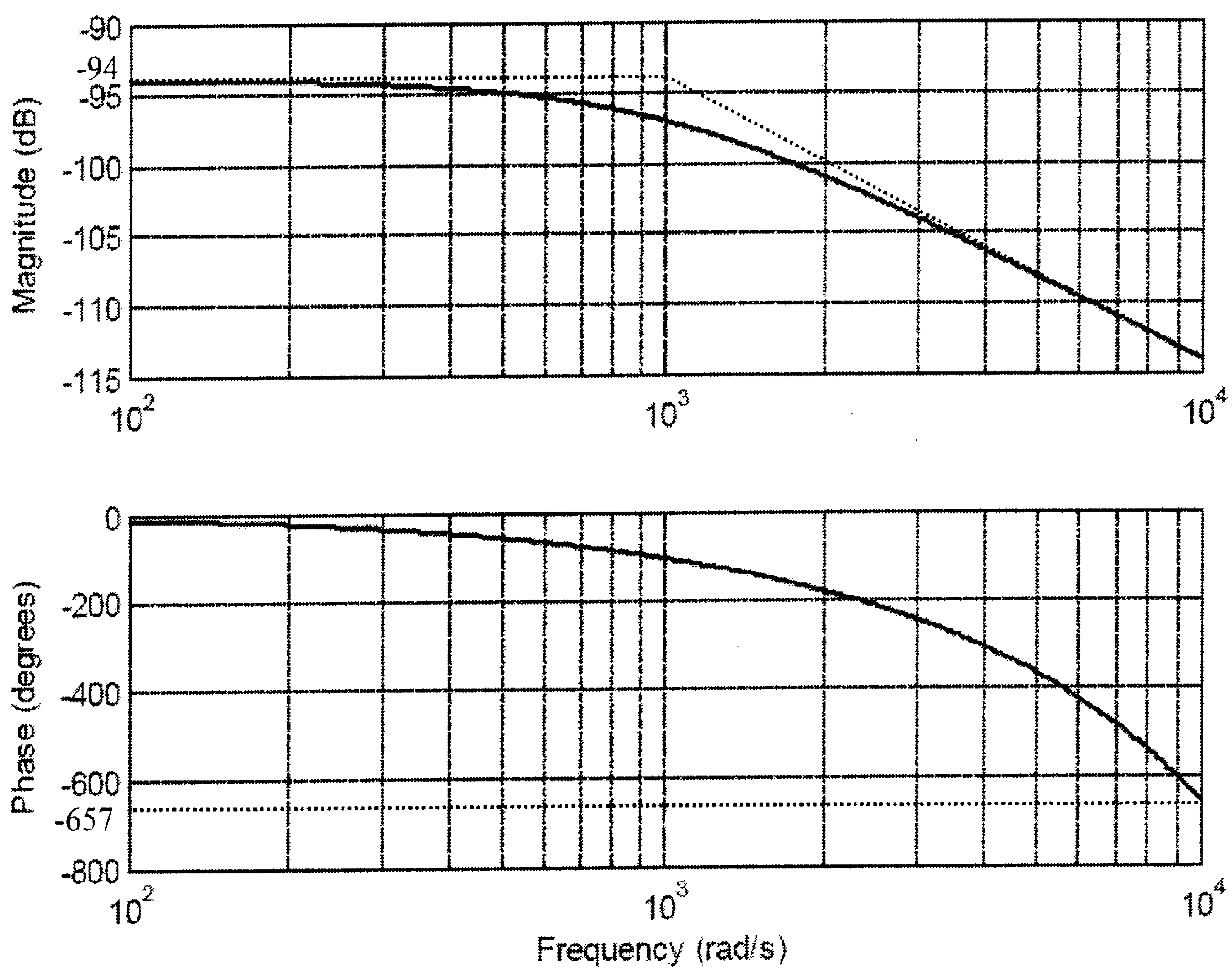

Figure 1

Q2 A third-order plant is described by the transfer function:

$$G_p(s)=\frac{6}{(s+1)(s+2)(s+3)}$$

is controlled by a *proportional-plus-integral* scheme: $\{k_p, k_i\}$. When $k_i = 3k_p$, the less-dominant plant pole is cancelled, such that the closed-loop characteristic equation is a function of k_p and is written as

$$1+6k_p\frac{1}{s(s+1)(s+2)}=0$$

The roots' loci is drawn in Figure 2.

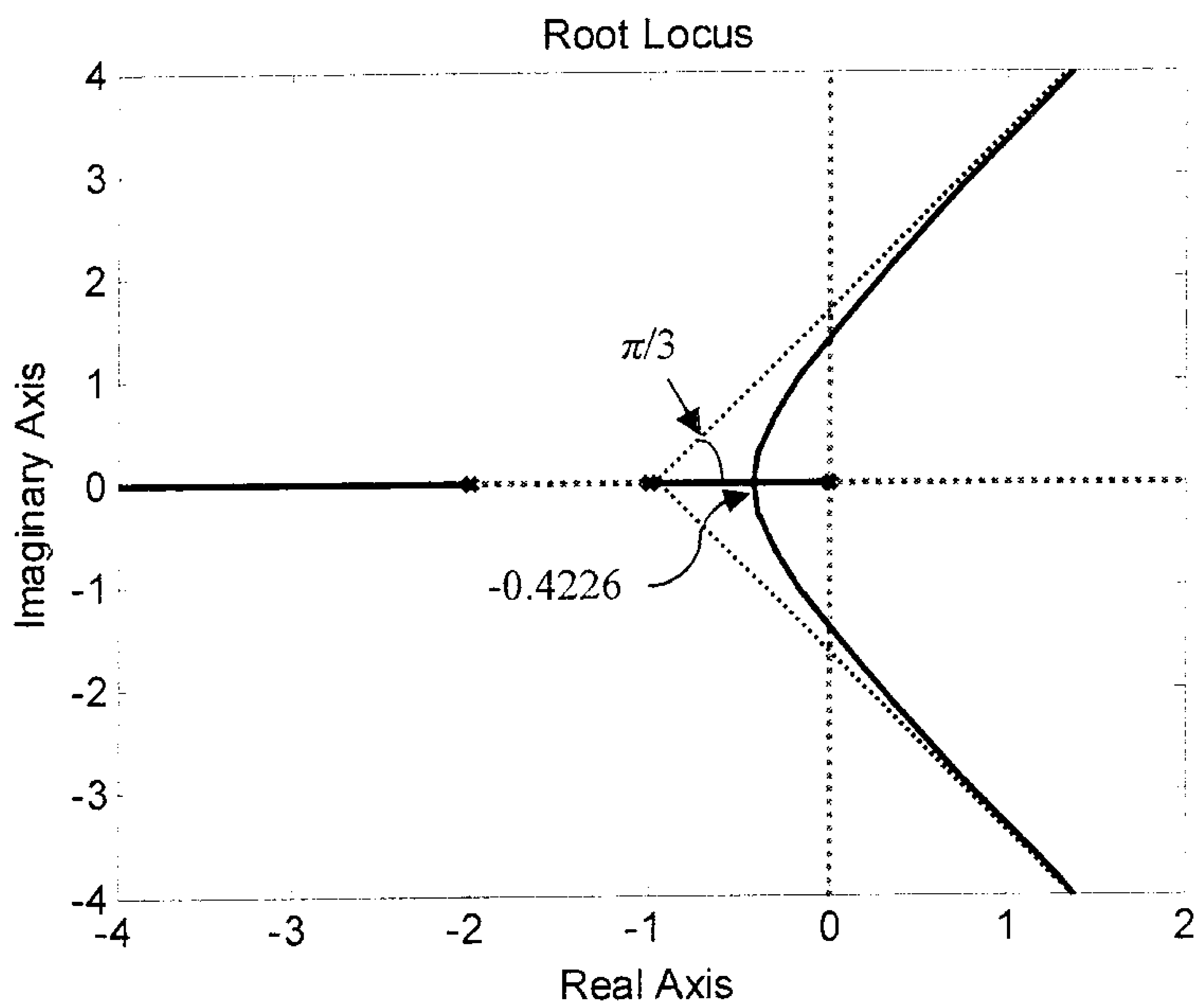

Figure 2

(a) Use Figure 2 to determine the controller gains k_p and k_i that yield the fastest possible non-oscillatory response. (4%)

(b) Comment on the general characteristics of the closed-loop step response, as k_p increases upwards from zero. (5%)

(c) Using the point of intersection of the asymptotes with the imaginary axis from Figure 2, estimate the limiting value of k_p for closed-loop stability. (4%)

(d) Would you expect this value of k_p determined from (c) to be an over-estimate or under-estimate? (2%)

(e) Use the Routh criterion to determine an accurate value/limit of k_p for closed-loop stability. (5%)

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Q3 A plant has transfer function:

$$G_p(s) = \frac{25}{(s+5)^3}$$

is controlled using a lag controller.

$$D(s) = \frac{k(1+Ts)}{(1+\alpha Ts)}$$

(a) Write down the open loop transfer function (with the lag controller). (3%)

(b) Take $\alpha = 20$ and find k and T, such that the low frequency gain of the open loop transfer function is 10 (20dB) and the controller zero cancels one of the plant poles. Show that the resultant open loop transfer function (OLTF) is

$$G_p(s) = \frac{62.5}{(s+5)^2(s+0.25)}$$

(7%)

(c) Use Figure 3 to estimate the gain margin (in dB) of the OLTF. (5%)

(d) Use Figure 3 to estimate the phase margin (in degree) of the OLTF. (5%)

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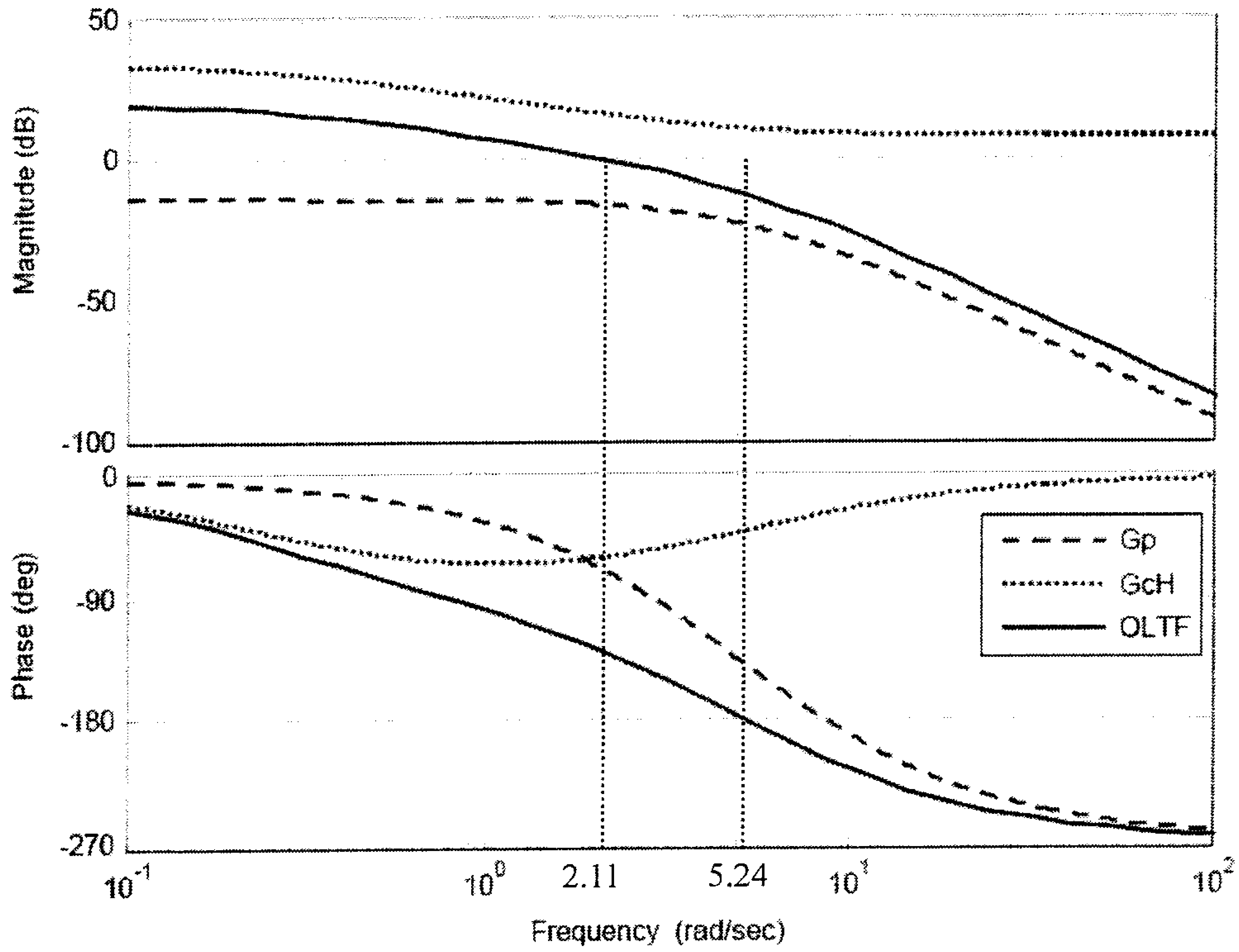


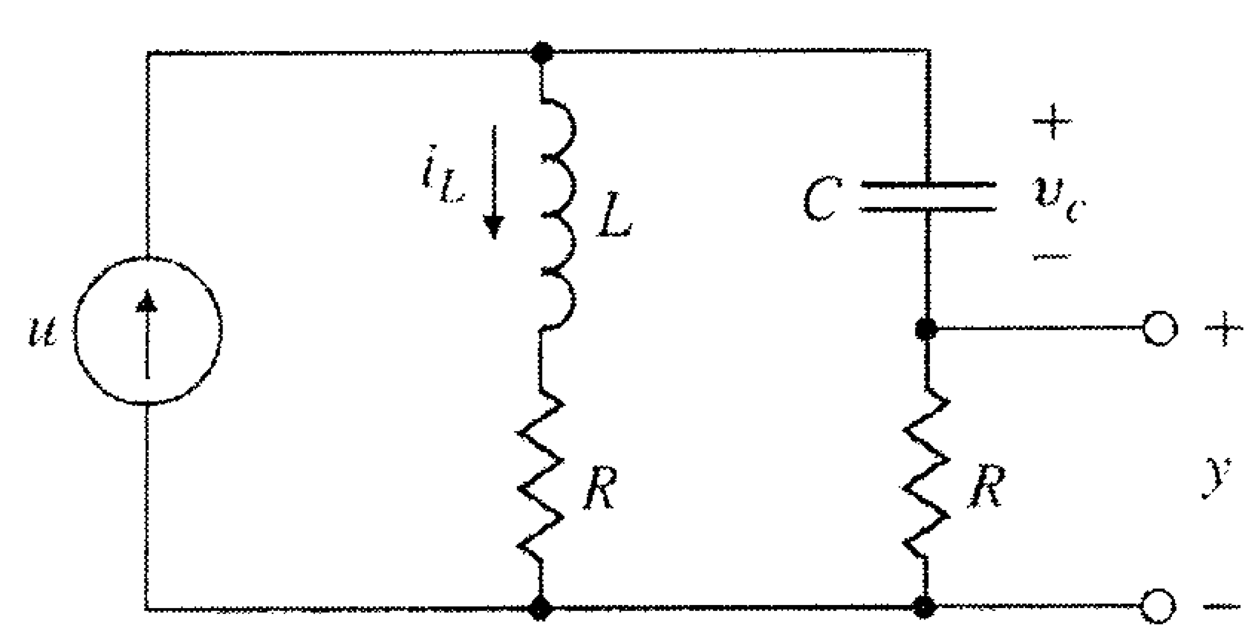
Figure 3

Q4 Consider the electric circuit shown in Figure 4.

(a) Write the internal (state) equations for the circuit. The input $u(t)$ is a current, and the output y is a voltage. Let $x_1 = i_L$ and $x_2 = v_c$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = F \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Gu$$

$$y = H \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Ju$$



(Find matrices F, G, H, and J) (5%)

Figure 4

(b) What condition(s) on R, L, and C will guarantee that the system is observable? (5%)

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Q5 The block diagram of a feedback system is shown in Figure 5.

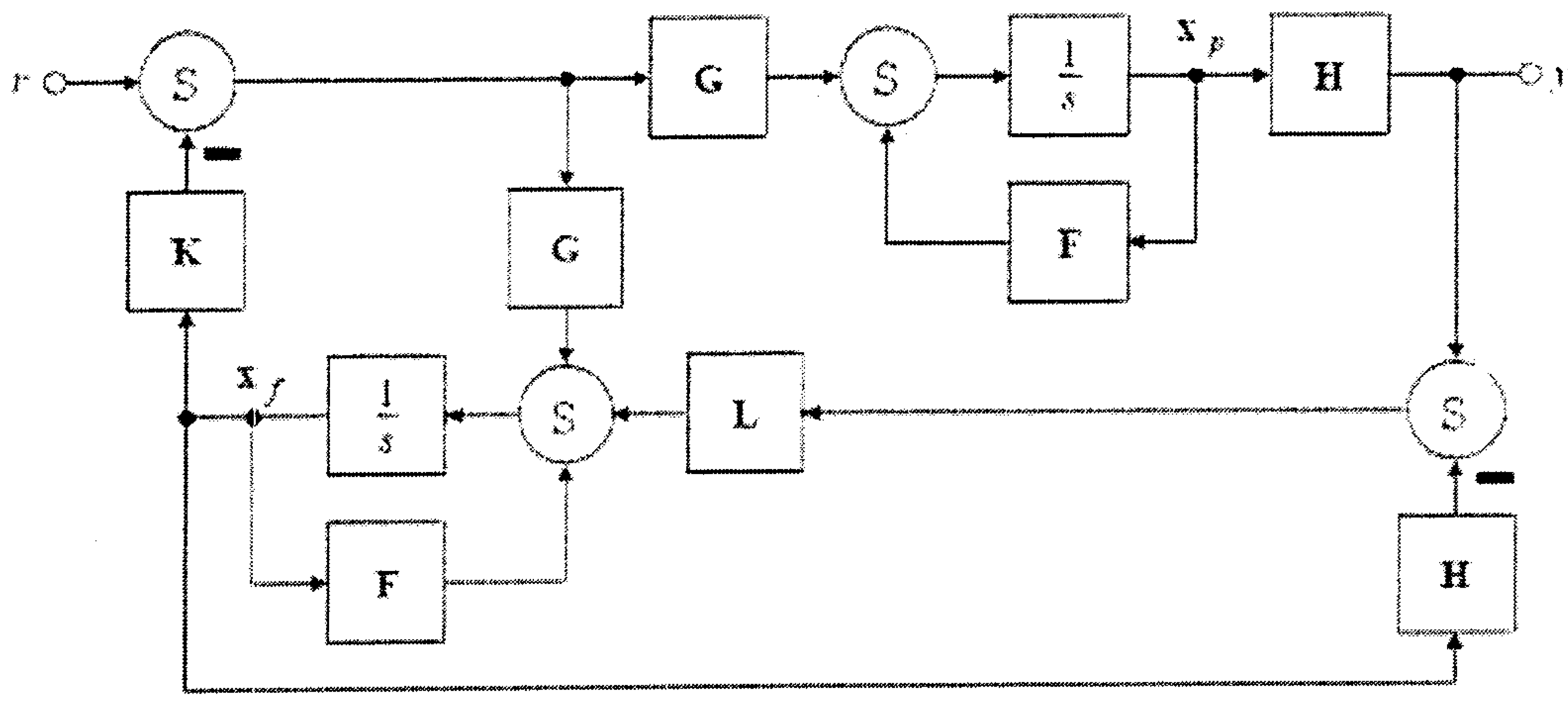

Figure 5

The system states are $x=\begin{bmatrix} x_p \\ x_f \end{bmatrix}$. The dimensions of the matrices are as follows:

F = $n\times n$; L = $n\times 1$; G = $n\times 1$; x = $2n\times 1$;
H = $1\times n$; r = 1×1 ; K = $1\times n$; y = 1×1 ;

(a) Write state equations $\dot{x} = Ax + Br$ for the system. (Find A and B) (5%)

(b) Let x = Tz, where $T=\begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}$. Show the system matrices $\begin{matrix} \dot{z} = A_c z + B_c r \\ y = C_c z \end{matrix}$

(Find A_c, B_c, and C_c) (5%)

(c) Is the system controllable? Please address your reason. (5%)

Q6 For the following system (Figure 6) with K=1, the Bode Plot is shown in Fig. 7.

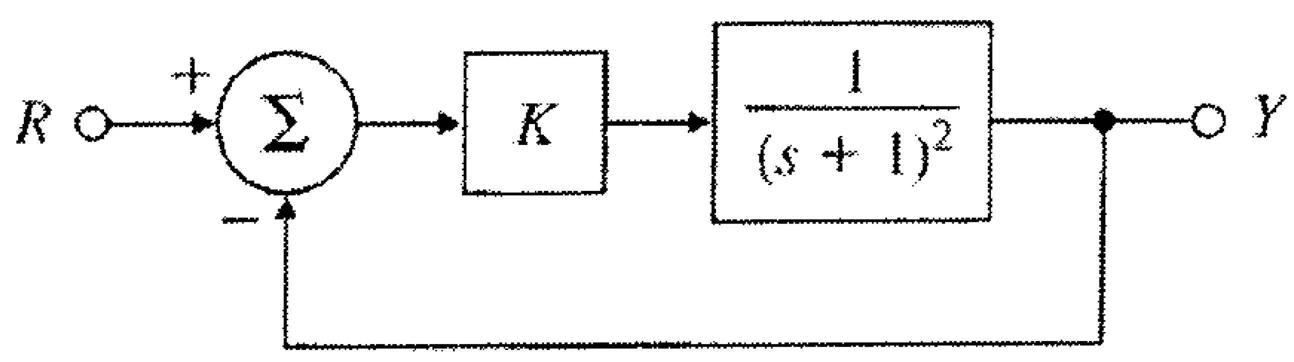

Figure 6

(a) Use bode plot information to draw the Nyquist plot (5%)

(b) Use Nyquist criterion to determine the stability of closed-loop system(5%)

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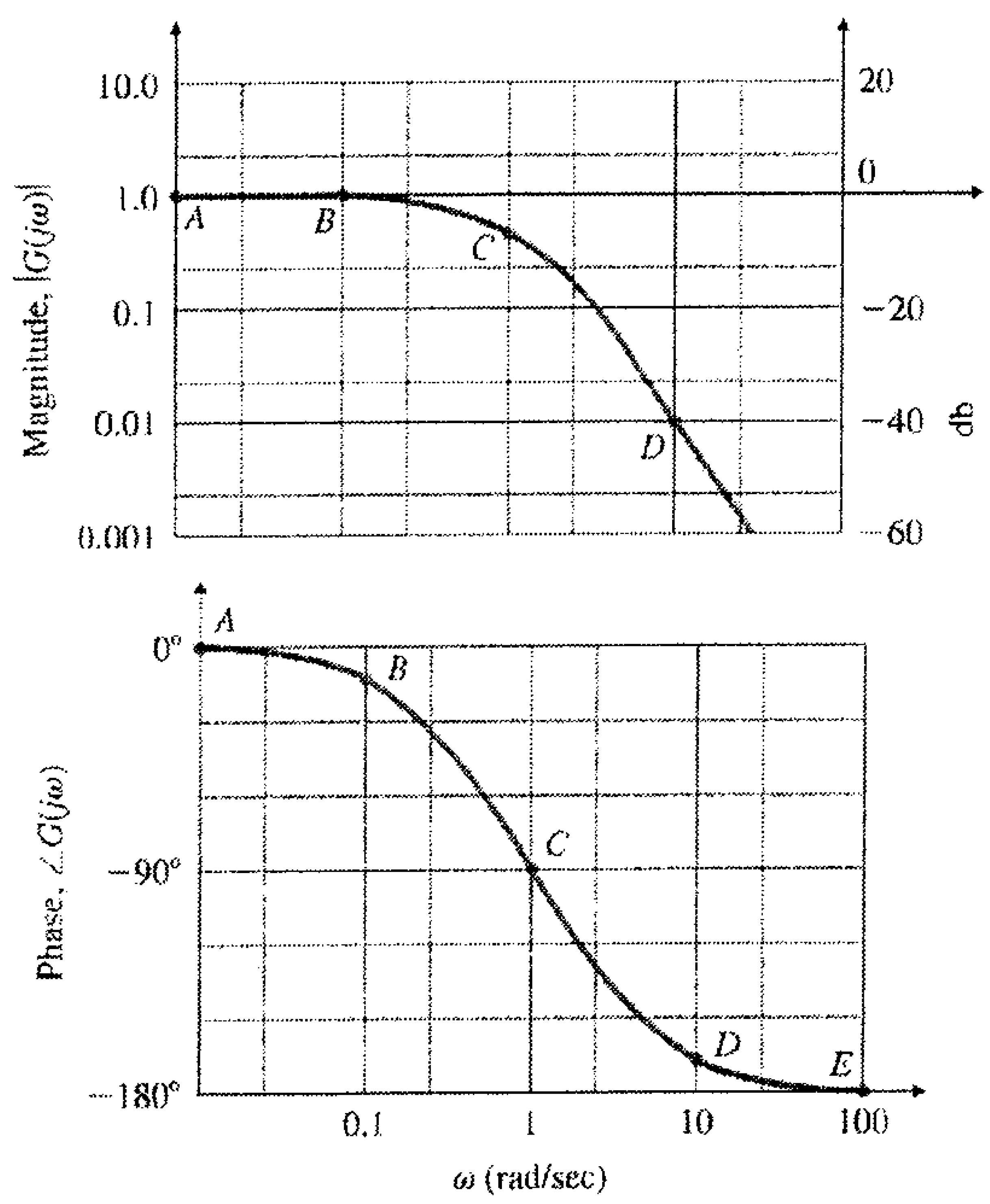

Figure 7

Q7 Suppose a DC drive motor with motor current u is connected to the wheels of a cart in order to control the movement of an inverted pendulum mounted on the cart. The linearized and normalized equations of motion corresponding to this system can be put in the form

$$\ddot{\theta} = \theta + v + u$$
$$\dot{v} = \theta - v - u$$

where θ = angle of the pendulum, and v = velocity of the cart.

(a) We wish to control θ by feedback to u of the form, $u = -K_1\theta - K_2\dot{\theta} - K_3 v$

Find the feedback gains $[K_1 \quad K_2 \quad K_3]$ so that the resulting closed-loop poles are located at -1, $-1 \pm j\sqrt{3}$ (5%)

(b) Assume that θ and v are measured. Construct an estimator for θ and $\dot{\theta}$ of the form

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$\dot{\hat{x}} = F\hat{x} + L(y - \hat{y})$ where $x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$ and $y = \theta$. Treat both v and u as known. Select $L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$ so that the estimator poles are at - 2 and - 2. (5%)

(c) What is the transfer function of compensator (the controller obtained by combining parts (a) through (b)) (5%)