國立政治大學 110 學年度碩士班暨碩士在職專班招生考試試題

考試科目	統計學	系所別	國際經營與貿易學系/國際經濟、國際財管、國際企管與行銷組一般生	考試時間	2 月 5 日(五)第 四 節

選擇題請在答案卡上作答，否則不予計分。

Multiple Choice Problems, choose the best answer (4 points each, 100 points in total)

1. Consider n samples independently drawn from a distribution with mean 0 and variance 1. What is the minimum value of n that will make variance of the sample mean no greater than 0.01?
   a. 30; b. 4; c. 100; d. n does not matter here; e. None of the above is correct.

2. A 95% confidence interval of a population mean is $[\underline{\mu}, \overline{\mu}]$. Which of the following statement is the most correct one?
   a. If random samples were drawn again and again, with $\underline{\mu}$ and $\overline{\mu}$ computed each time, the population mean would lie in the interval $[\overline{\mu}, \underline{\mu}]$ 95% of the samples.
   b. If $\underline{\mu} = 0.1$ and $\overline{\mu} = 0.5$, the population mean would lie in the interval $[0.1, 0.5]$ with 95% probability.
   c. If $\underline{\mu} = 0.05$ and $\overline{\mu} = 0.55$, the population mean might still lie outside the interval $[0.05, 0.55]$.
   d. If $\underline{\mu} = 0.03$ and $\overline{\mu} = 0.56$, the population mean would lie in the interval $[0.03, 0.56]$ greater than 95% probability.
   e. a and c.

3. If $E[X|Y] = E[X]$, then
   a. X and Y are mutually independent; b. X and Y are uncorrelated; c. $E[X^2|Y] = E[X^2]$; d. $Var(X|Y) = Var(X)$; e. $E[X] = 0$.

4. Consider the following hypothesis test about population mean: $H_0 : \mu = 3$, $H_1 : \mu > 3$. Which of the following results gives most supportive evidence to reject the hypothesis?
   a. Sample mean equals to 5 and standard error equals to 1.
   b. Sample mean equals to 5 and standard error equals to 2.
   c. Sample mean equals to 3.5 and standard error equals to 1.
   d. Sample mean equals to 2 and standard error equals to 1.
   e. Sample mean equals to 2 and standard error equals to 0.5.

5. Suppose we want to test $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$:
   a. The power function of the test is a function of μ and describes the probability of rejecting H_0.
   b. The maximum probability of committing a Type I error is called the power of the test.
   c. The minimum probability of committing a Type II error is called the size of the test.
   d. When the power function is evaluated at μ_0, the value equals to the probability of committing a Type II error.
   e. None of the above is correct.

6. Consider two continuous random variables (X, Y) with $X \in [0, \infty]$ and $Y \in [1, 5]$. The joint density function of (X, Y) is

$$f_{XY}(x, y) = \begin{cases} \frac{1}{4} y \exp(-xy) & \text{if } X \in [0, \infty], Y \in [1, 5] \\ 0 & \text{otherwise} \end{cases}$$

   What is the conditional density function of X given $Y = 1$ if $X \in [0, \infty]$?
   a. $\exp\left(-\frac{x}{2}\right)$; b. $\frac{1}{2}\exp(-x)$; c. $\exp(-x)$; d. $\frac{1}{4}\exp(-x)$; e. $\frac{1}{4}\exp\left(-\frac{1}{2}x\right)$.

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7. Consider an experiment having two outcomes: success with probability $p = 0.12$ and failure with probability $1 - p = 0.88$. Suppose we independently conduct the experiment n times and estimate p with

$$\hat{p} = \frac{\text{number of success experiments}}{n}$$

What is the minimum value of n that will make standard deviation of $\hat{p}$ no greater than 0.01?
a. 30; b. 1056; c. 33; d. n does not matter here, e. None of the above is correct.

8. A population is composed of 5 numbers: 2, 4, 6, 8, 10. Let μ and σ denote mean and standard deviation of the population, and $\mu_{\bar{X}}$ and $\sigma_{\bar{X}}$ denote mean and standard deviation of the sample mean from the population. Which of the following statement is the most correct one?
a. $\mu = 6, \sigma = 3.16$; b. $\mu_{\bar{X}} = 6, \sigma_{\bar{X}} = 3.16$; c. $\mu_{\bar{X}} = 6, \sigma_{\bar{X}} = 2.83$; d. $\mu = 6, \sigma = 2.83$; e. $\mu_{\bar{X}} = \mu$, $\sigma = 3.16$.

9. If we calculate sample mean from the population in question 8, with sample size equal to 2. Let $f_{\bar{X}_2}(x)$ and $F_{\bar{X}_2}(x)$ denote the theoretical density and cumulative distributions of the sample mean, respectively. Which of the following statement is the most correct one?
a. $f_{\bar{X}_2}(4) = 0.13$, $F_{\bar{X}_2}(4) = 0.2$; b. $f_{\bar{X}_2}(5) = 0.2$, $F_{\bar{X}_2}(4) = 0.2$; c. $f_{\bar{X}_2}(3) = 0.1$, $F_{\bar{X}_2}(4) = 0.3$; d. $f_{\bar{X}_2}(7.8) = 0.1$, $F_{\bar{X}_2}(8) = 0.7$; e. $f_{\bar{X}_2}(5.5) = 0.08$, $F_{\bar{X}_2}(7) = 0.7$.

10. If we calculate a sample mean from the population in question 8, with sample size equal to 2. Let $\mu_{\bar{X}_2}$ and $\sigma_{\bar{X}_2}$ denote mean and standard deviation of the sample mean. Which of the following statement is the most correct one?
a. $\mu_{\bar{X}_2} = 5.5$, $\sigma_{\bar{X}_2} = 3.16$; b. $\mu_{\bar{X}_2} = 6$, $\sigma_{\bar{X}_2} = 1.73$; c. $\mu_{\bar{X}_2} = 6$, $\sigma_{\bar{X}_2} = 2.83$; d. $\mu_{\bar{X}_2} = 5.5$, $\sigma_{\bar{X}_2} = 1.89$; e. $\mu_{\bar{X}_2} = 6$, $\sigma_{\bar{X}_2} = 2.04$.

11. Consider two discrete random variables (X, Y) with $X \in \{0, 1, 2\}$ and $Y \in \{0, 3\}$. The joint probability mass function of (X, Y) is

$$p_{XY}(x, y) = \begin{cases} \frac{1}{3} & \text{if } x = 0, y = 0 \\ \frac{1}{3} & \text{if } x = 1, y = 3 \\ \frac{1}{3} & \text{if } x = 2, y = 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the conditional expectation of X given $Y = 0$?
a. 0.5; b. 1; c. $\frac{5}{3}$; d. $\frac{2}{3}$; e. None of the above.

12. Let X be a positive random variable whose expectation equals to 5. Consider the probabilities that: $P(X < 10)$ and $P(X \geq 8)$. Which of the following statement is the most correct one?
a. $P(X \geq 8) > 0.7$; b. $P(X < 10) > 0.5$; c. $P(X < 10) < 0.3$; d. $P(X < 10) < 0.4$; e. $P(X < 8) < 0.2$.

13. Suppose that $\Omega = \{a, b, c, d, e\}$ and A and B are subsets of Ω. It is known that the complement of A, $A^c = \{a, b, c\}$ and the complement of B, $B^c = \{b, c, d\}$. What is the complement of A or B, $(A \cup B)^c$?
a. $\{d\}$; b. $\{b, c\}$; c. $\{a, e\}$; d. $\{a, b, c, d\}$; e. $\{a, d, e\}$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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14. Let X be a continuous random variable with density function

$$f_X(x) = \begin{cases} \frac{3}{8}x^2 & \text{if } x \in [0,2] \\ 0 & \text{otherwise} \end{cases}$$

Let $Y = \sqrt{X+1}$. What is the density function of Y if $Y \in [1, \sqrt{3}]$?
a. $\frac{3}{8}(y^2-1)^2 y$; b. $\frac{3}{4}(y^2-1)^2 y$; c. $\frac{3}{8}(y-1)^2 y$; d. $\frac{3}{4}(y^2-\sqrt{3})^2 y$; e. $\frac{3}{4}(y-1)^2$.

15. Consider n samples independently drawn from a distribution with mean 0 and variance 1. What is the minimum value of n that will make bias of the sample mean no greater than 0.1?
a. 30; b. 3; c. 100; d. n does not matter here; e. None of the above is correct.

16. Let X be a random variable whose expectation equals to 3 and variance equals to 5. Consider the probabilities: $P(|X-3|<10)$ and $P(|X-3|\geq 8)$. Which of the following statement is not the correct one?
a. $P(|X-3|<10) > 0.75$; b. $P(|X-3|\geq 8) \geq 0.1$; c. $P(|X-3|\geq 8) < 0.05$; d. $P(|X-3|\geq 8) \geq 0.06$; e. $P(|X-3|<10) > 0.8$.

17. Consider n samples $(X_1, X_2, \ldots, X_n)$ independently drawn from a distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$. The following estimator is used to estimate σ^2:

$$\widehat{\sigma_n^2} = \frac{1}{n}\sum_{i=1}^{n} X_i^2.$$

What is the minimum value of n that makes bias of $\widehat{\sigma_n^2}$ no greater than 0.01?
a. 30; b. 7; c. $\widehat{\sigma_n^2}$ is a biased estimator and so increasing n has no effect on reducing the bias; d. n does not matter here; e. $\widehat{\sigma_n^2}$ is a biased estimator in finite sample but increasing n can reduce the bias.

18. Suppose Y is a Poisson random variable with mean $\lambda = 4$ and X is a Chi-square random variable with degree of freedom $df = 2$. Which of the following statement is the most correct one?
a. Covariance between X and Y is greater than 6; b. $E[X+Y] = 4$; c. Variance of $X+Y$ equals to 8; d. Covariance between $2X$ and $3Y$ is between -24 and 24; e. Variance of $X+Y$ is between 8 and 16.

19. In country A, according to past experience, 40% of video game consoles sold were PS4, 10% were xbox I and 50% were Switch. To maintain the inventory of each type of video game consoles, Tony conducts a survey and gather a random sample of 100 sales of video game consoles and finds that 30 of them are PS4, 30 are xbox I and 40 are switch. To test whether past pattern of sales of video game consoles still prevails, Tony uses the chi-square test with 1% level of significance. Let $Q_{\chi_k^2}(\alpha)$ denote the αth quantile of a chi-square random variable with degree of freedom k:
a. The value of the test statistic is 44.5 and the critical value used should be $Q_{\chi_2^2}(0.99)$.
b. The value of the test statistic is 17.83 and the critical value used should be $Q_{\chi_3^2}(0.995)$.
c. The value of the test statistic is 42 and the critical value used should be $Q_{\chi_2^2}(0.99)$.
d. The value of the test statistic is 45.33 and the critical value used should be $Q_{\chi_3^2}(0.99)$.
e. The value of the test statistic is 45.33 and the critical value used should be $Q_{\chi_2^2}(0.995)$.

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20. Let (X, Y) be two random variables such that standard deviation of X is 2 and covariance between X and Y is 2. What is the covariance between $3X$ and $X + 3Y$?
a. 21; b. 10; c. 25; d. 16; e. 30.

21. Let X be a strictly positive random variable, whose expectation equals to 0.5 and variance equals to 1. Which of the following statement is the most correct one?
a. $E[\ln(2X)] > 0.5$; b. $E[\ln(2X)] \leq 0$; c. $\ln(E[X^2]) > -0.2$; d. $\ln(E[X^2]) < -0.3$; e. $2\ln(E[X]) > -1$.

22. A manager wants to see whether the proportion of acceptable products from supplier A, p_A is greater than for supplier B, p_B. The manager draw $n_A = 123$ random sample from A's and $n_B = 94$ samples from B's products. She finds that acceptable rates of samples for products from A and B are $\bar{p}_A = 0.87$ and $\bar{p}_B = 0.79$, respectively. The manager plans to compute a test statistic and compare it with critical value from a standard normal distribution with significance level 5%:
a. The value of the test statistic is 1.575 and the critical value is 1.645.
b. The value of the test statistic is 3.061 and the critical value is 2.33.
c. The value of the test statistic is 0.984 and the critical value is 1.96.
d. The value of the test statistic is 3.33 and the critical value is 2.33.
e. The value of the test statistic is 1.775 and the critical value is 1.645.

23. Consider an experiment having two outcomes: success (probability $p = 0.1$) and failure (probability $1 - p = 0.9$). Suppose the experiment is independently conducted n times and p is estimated with

$$\hat{p} = \frac{\text{number of success experiments}}{n}$$

What is the minimum value of n that will make $E[\hat{p}] - p \leq 0.001$?
a. 30; b. 6; c. 90; d. n does not matter here; e. None of the above is correct.

24. Which of the following statement is the most correct one?
a. A Type I error is an error such that if the null is hypothesis is true, we reject the null.
b. A Type II error is an error such that if the null is hypothesis is true, we reject the null.
c. A Type II error is an error such that if the null is hypothesis is not true, we do not reject the null.
d. A Type I error is an error such that if the null is hypothesis is not true, we do not reject the null.
e. a and c.

25. In country B, men's longevity are investigated and the following results are found: 1. Probability that a man lives at least 70 years is 77%; 2. Probability that a man lives at least 80 years is 48%. What is the conditional probability that a man's age is at least 80 years old given that he has just celebrated his 70th birthday?
a. $\frac{48}{77}$; b. 0.48; c. 0.77; d. 1; e. None of the above is correct.

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