1. (28%) Consider the two-mass system in Figure 1, where k_1 and k_2 are spring constants; x_1 and x_2 are displacements of m_1 and m_2, assuming that both springs have their natural lengths when $x_1 = x_2 = 0$. The surface is frictionless and f is an external force applied to m_1.
   (a) (6%) Choose x_1 as the output of the system. Write down the state space representation using $\mathbf{x} = [x_1, x_2, \dot{x}_1, \dot{x}_2]^T$ as the state vector, where $\dot{x}_i$ denotes the time derivative of x_i for $i = 1,2$.
   (b) (8%) Find the transfer function from f to x_1.
   (c) (6%) Suppose that $\frac{k_1}{m_1} = \frac{k_2}{m_2} = r$ and $\frac{k_2}{m_1} = \frac{r}{6}$. Find the resonant frequencies of the system in terms of r.
   (d) (8%) Let $f(t) = \sin(\omega_0 t)$ and $\dot{x}_1(0) = 0$. Find ω_0, $x_2(0)$, and $\dot{x}_2(0)$ such that $x_1(t) = 0$ for all $t \geq 0$.

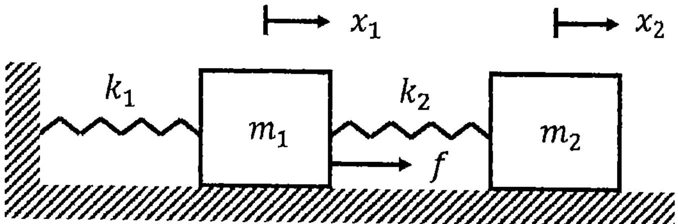


Figure 1: Two-mass system

2. (22%) Consider the feedback control system in Figure 2, where

$$G(s) = \frac{10}{(s+1)(s+10)}, \qquad C(s) = \frac{k(as+1)}{s}, \quad k, a > 0$$

   (a) (6%) Find the conditions on k and a such that the closed-loop system is stable.
   (b) (8%) Find k and a to satisfy the following requirements simultaneously:
      i. The closed-loop transfer function from r to y is a stable, 2^{nd} order rational function.
      ii. The steady-state error with respect to the unit-ramp input $r(t) = t$, $t \geq 0$, is less than 0.2.
      iii. The percent maximum overshoot with respect to the unit-step input $r(t) = 1$, $t \geq 0$, is less than 10%.
   (c) (8%) Let $a = \frac{1}{20}$. Draw the root locus when k increases from 0 to ∞. If $k = 1$, what is the gain margin of the system?

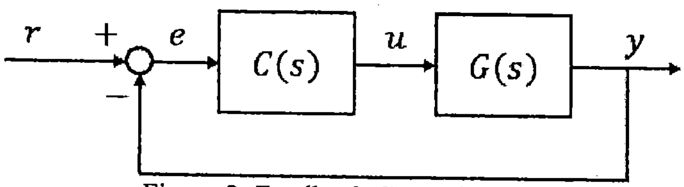
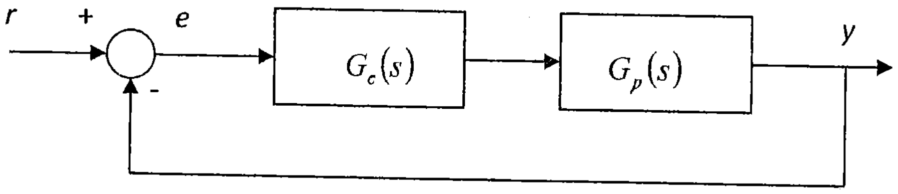


Figure 2: Feedback Control System

注意:背面有試題

3. (32%) Consider the following feedback system with plant $G_p(s)$ and controller $G_c(s)$.



Let $G_p(s)=\frac{s^3}{s+2}$, where $G_p(j\omega)=\frac{-\omega^4-2j\omega^3}{\omega^2+4}$. The controller $G_c(s)=K$.

(a) (6%) Plot the root locus for $K>0$ and $K<0$, respectively.

(b) (5%) Sketch the Nyquist plot for $K>0$.

(c) (4%) Analyze the stability from Nyquist plot for all K.

(d) (6%) To stabilize the system, a control engineer chooses $G_c(s)=\frac{K}{s(s+p)}$, where $p>0$. Show that the root locus contains a circle for $K>0$.

(e) (4%) As in part (d), design the closed-loop poles at $-1\pm j$, design the controller $G_c(s)$

(f) (3%) From the root locus in part (d), what are the Gain margin?

(g) (4%) In part (e) and input $r(t)=1+\cos\left(\sqrt{2}t-30^\circ\right)$, what is the steady state output?

4. (18%) Let the state equations of an LTI system be $\dot{x}(t)=Ax(t)+Bu(t)$, and $y(t)=Cx(t)$ and $x(t)=\phi(t)x(0)+\int_0^t\phi(t-\tau)Bu(\tau)d\tau$, where $\phi(t)$ is the state transition matrix and initial condition is $x(0)$.

(a) (3%) What is the impulse response, $h(t)$, of the output without initial condition?

(b) (6%) Consider a system ruled by $\ddot{y}(t)+3\dot{y}(t)+2y(t)=2\dot{u}(t)+3u(t)$, find the state space representation of A, B, and C with Controllability Canonical Form.

(c) (4%) Without initial condition, what is the impulse response of the output by part (a)? Hint: the Laplace transform of $\phi(t)$ is $[sI-A]^{-1}$. Do NOT compute $\phi(t)$ directly.

(d) (5%) Solve the impulse response, $h(t)$, by $\ddot{h}(t)+3\dot{h}(t)+2h(t)=2\dot{\delta}(t)+3\delta(t)$ with differential equation approach. What is your observation with the result in part (c)?