※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

(一)單選題：共 8 題，每題 5 分，答錯倒扣 5/4 分，倒扣至該大題(即單選題) 0 分為止

1. For a metal with specified work function $\Phi_0 = 3.1$ eV subject to exposure of light with various wavelengths and intensity of 8.01 W/m^2 directed at the metal sphere with a radius of 5 *cm*

(A) The maximum wavelength required to stimulate the photoelectron emission is $\lambda_{max} = 350$ *nm*, which is the kind of ultra-violate (UV)

(B) Under the exposure of light with $\lambda = 248$ *nm*, the maximum energy of the photoelectrons emitted from the metal is 1.5 eV

(C) Under the exposure of light with $\lambda = 155$ *nm*, the maximum energy of the photoelectrons emitted from the metal is 4.5 eV

(D) Under the condition that the exposure of light $\lambda = 248$ *nm* and 0.25% of the incident photons create photoelectrons, the number of emitted photoelectrons is 7.854×10^{14}

(E) The increase of exposure light intensity can increase the photoelectrons numbers and energy

2. Which of the following statements is **wrong**, in regard to the physics of a 1D oscillator in the quadratic potential energy $V(x) = 1/2\ k\ (x-x_{min})^2$? Let *n* denote the state quantum number of oscillation, with *n* = 0 for the ground state and *n* = 1, 2, … for excited states.

(A) The classical ground state energy is given by $E_0 = 0$.

(B) Quantum energy levels $\{E_n' s\}$ are equally spaced.

(C) Oscillation frequencies are identical for classical and quantum oscillators.

(D) For large *n*, the quantum description approaches the classical one.

(E) The ground state wave function $\Phi_0 = 0$ for $|x| > x_t$. (Note: x_t is the classical turning point where the kinetic energy $E_0 - V(x_t)$ vanishes.)

3. Assuming that the conduction electrons in a cube of a metal on edge 1 *cm* behave as a free quantized gas, what is the number of states which are available in the energy interval 4.00-4.01 eV, per unit volume?

(A) 1.356×10^{18}

(B) 1.356×10^{20}

(C) 1.356×10^{22}

(D) 8.478×10^{25}

(E) 8.478×10^{28}

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4. What is the depletion-layer width for a *p-n* junction with zero bias in germanium, given that the impurity concentrations are $N_A = 1\times10^{23}\ m^{-3}$ and $N_D = 2\times10^{22}\ m^{-3}$, respectively, at $T = 300$ K, relative permittivity $\varepsilon_r = 16$ and contact potential difference $V_0 = 0.8$ V?

(A) 0.0725 μm

(B) 0.145 μm

(C) 0.29 μm

(D) 0.58 μm

(E) 1.16 μm

5. For a muon with $m_\mu = 200\ m_e$ captured by a proton to form a muonic atom, please calculate the orbital radius, wavelength, speed, and energies at ground state $n = 1$: r_1, λ_1, v_1, E_1, and photon wavelength λ_α for transition from E_2 to E_1 (α-line)

(A) $r_1 = 3.65\times10^{-13}\ m = 0.365\ pm$

(B) $\lambda_1 = 2.056\times10^{-12}\ m = 2.056\ pm$

(C) $v_1 = 2.1872\times10^{6}\ m/s$

(D) $E_1 = -2505$ eV

(E) $\lambda_\alpha = 6.74\times10^{-9}\ m = 6.74\ nm$

6. Consider a particle moving in a 2D square quantum well, with well width = L and well depth = ∞. Take the potential well bottom = 0. Which of the following statements is **correct**, in regard to the quantum state?

(A) The ground state energy is four times as large as that in a 1D quantum well of identical width and depth.

(B) The ground state energy is 1/4 of the first excited state energy.

(C) The ground state wave function is given by the sum $F_0(x) + F_0(y)$, ((x,y) = particle position), where F_0 is the ground state wave function in a 1D quantum well of identical width and depth.

(D) The ground state wave function vanishes at the well center.

(E) The ground state wave function is even with respect to the inversion operation (x,y) → (-x,-y).

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7. Electrons with energy of 2 eV are separately incident on 10 eV barriers with 0.5 nm and 1 nm wide. What are their respective transmission probabilities ($T_{\text{in 0.5 nm barrier width}}$, $T_{\text{in 1 nm barrier width}}$)?
(A) (1.1×10^{-7}, 2.4×10^{-7})
(B) (1.1×10^{-7}, 1.3×10^{-14})
(C) (1.3×10^{-14}, 5.1×10^{-14})
(D) (2.4×10^{-7}, 1.3×10^{-14})
(E) (2.4×10^{-7}, 5.1×10^{-14})

8. In terms of a particle trapped in a box with the length of L, which of following statements is **wrong**?
(A) The normalized wave functions of the particle are $\psi_n = \sqrt{\frac{2}{L}} sin\frac{n\pi x}{L}$
(B) At ground state, the probability of the particle which is found between 0.45 L and 0.55 L is about 10%.
(C) At first excited state, the probability of the particle which is found between 0.45 L and 0.55 L is about 0.65%.
(D) At first excited state, the probability of the particle which is found at $x = 0.5\ L$ is zero.
(E) At a particular place in the box, the probability of the particle may be different for different quantum numbers.

(二)複選題：共 8 題，每題 5 分，答錯倒扣 1 分，倒扣至該大題（即複選題）0 分為止

9. Which of the following statements are **correct** concerning the quantum energy eigenstate of a free particle? Let E = energy, **p** = momentum, and **k** = wave vector of the state.
(A) E's of all eigenstates form a continuous spectrum.
(B) $2E_c/|\mathbf{p}_c|$ gives the speed of a moving wave packet formed by mixing the eigenstates with (E, **p**) around (E_c, $\mathbf{p}_c$).
(C) A particle with **k** will behave approximately classically, when hitting a wall with an opening of the size $L >> 1/|\mathbf{k}|$.
(D) A mixed state formed of two eigenstates such as $\exp(i\mathbf{k}_1\cdot\mathbf{r})\exp(-iE_1t) + \exp(i\mathbf{k}_2\cdot\mathbf{r})\exp(-iE_2t)$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ differ but $|\mathbf{k}_1| = |\mathbf{k}_2|$, is not an eigenstate of momentum but is that of energy.
(E) If one measures the momentum of mixed state $\exp(i\mathbf{k}\cdot\mathbf{r}) + \exp(-i\mathbf{k}\cdot\mathbf{r})$ for many times by performing the measurement on a very large number of identical copies of the state, the distribution of measured values will have a width of the order of $|\mathbf{k}|$.

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10. Which of the following experimental observations that challenged classical physics around 1900?

(A) Blackbody radiation.

(B) Electromagnetism and the origin of magnetic fields, which did transform from one coordinate system to another.

(C) Line spectra of hydrogen and other atoms.

(D) The photoelectric effect.

(E) Compton scattering.

11. Consider the quantum system of two identical, spin-1/2 particles moving in a 1D infinite quantum well, in the presence of a uniform magnetic field **B**. Let x_1 and x_2 denote the two particle positions, s_{z1} and s_{z2} the two particle spins in the **B** direction, E_z = Zeeman energy, and dE = energy spacing between the ground and first excited states of one particle in the well at **B** = 0. Which of the following statements are correct?

(A) When **B** is turned off, the ground state of the two-particle system consists of two particles of opposite spins.

(B) When **B** is turned on, the ground state of the two-particle system consists of two particles of parallel spins, if $E_z >> dE$.

(C) At **B** = 0, the wave function of the two-particle system is of the determinant form $F(x_1, s_{z1})G(x_2, s_{z2}) - G(x_1, s_{z1})F(x_2, s_{z2})$.

(D) The Pauli exclusion principle implies that the wave function of the two-particle system must always vanish if $x_1 = x_2$ and $s_{z1} = s_{z2}$.

(E) All of the above are true.

12. A muon has a half lifetime $\tau_{1/2} = 1.56\times10^{-6}$ s and a mass of 200 m_e. This muon is moving with a speed of $\upsilon = 0.6$ c. Please calculate mean lifetime τ_0, the traveling distance *d* measured at rest, during mean lifetime τ_0 the relativistic momentum *P*, the de-Broglie wavelength λ_d, the total energy E and kinetic energy (*KE*) in the unit of eV, the wavelengths of a photon with the same kinetic energy λ_{KE} or the same momentum *P*, λ_p

(A) The mean lifetime and traveling distance measured at rest is $\tau_0 = 3.57\times10^{-6}$ s, $d = 786\ m$

(B) The relativistic momentum is $P = 76.66$ MeV/c

(C) The de-Broglie wavelength is $\lambda_d = 1.675\times10^{-15}\ m$

(D) The total energy and kinetic energies are $E = 165.77$ MeV, $KE = 35.55$ MeV

(E) The wavelengths of photon with the same *P* or *KE*: $\lambda_{KE} = 4.8524\times10^{-14}$ m, $\lambda_P = 1.6175\times10^{-14}\ m$

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13. An electron and a positron move side by side, at the same velocity of 0.8c, in the same direction on the x-axis. Finally, these two particles merge together thus annihilate each other, and two photons are generate

(A) These two photons move in the same direction
(B) These two photons move in opposite direction with momentum $p_1 = 2.55$ MeV/c, $p_2 = 0.51$ MeV/c
(C) These two photons move off with the energy $E_1 = 1.533$ MeV, $E_2 = 170.36$ KeV
(D) These two photons move in the same direction, with the energy $E_1 = 1.55$ MeV, $E_2 = 0.51$ MeV
(E) These two photons move off, with the wavelength $\lambda_1 = 8.087\times10^{-13}\ m$, $\lambda_2 = 7.278\times10^{-12}\ m$

14. Which of the following statements are **correct**?

(A) If the mean track length of 100 MeV π mesons is 4.88 *m* up to the point of decay, their mean lifetime is 1.17×10^{-8} s.
(B) A neutrino of energy 2 GeV collides with an electron. The maximum momentum transfer to the electron is 2.0437 GeV/c.
(C) A positron-electron pair production can occur in the interaction of a gamma ray with electron, via $\gamma + e^- \rightarrow e^- + e^+ + e^-$. The threshold is $4mc^2$.
(D) A linear accelerator produces a beam of excited carbon atoms of kinetic energy 120 MeV. Light emitted on de-excitation is viewed at right angles to the beam and has a wavelength λ'. If λ is the wavelength emitted by a stationary atom, the value of $(\lambda' - \lambda)/\lambda = 0.0099$. (Take the rest energies of both protons and neutrons to be 10^9 eV).
(E) All statements are correct.

15. An electron is in a box 0.1 nm across, which is the order of magnitude of atomic dimensions. Find its permitted energies.

(A) 19 eV
(B) 38 eV
(C) 76 eV
(D) 152 eV
(E) 608 eV

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16. Which of the following statements are **correct**?

(A) Possible values of the principle quantum number n are 1, 2, 3, …, determining the electron energy

(B) Possible values of the orbital quantum number l are 0, 1, 2, ….., n, determining the angular-momentum direction

(C) The electron spin direction can be described by spin magnetic quantum number m_s, which is independent with other quantum numbers.

(D) No two electrons in an atom can exist in the same quantum state. They must have a different set of quantum numbers.

(E) Electrons are fermions and can be described by the antisymmetric wave function, $\psi_F = \frac{1}{\sqrt{2}}[\psi_a(1)\psi_b(2) - \psi_a(2)\psi_b(1)]$

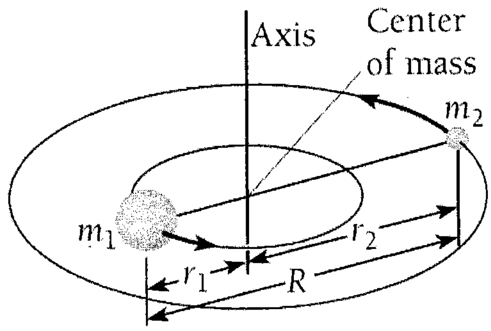
(三)非選擇題：共 2 題，每題 10 分

17. (10%) Derive the total energy of Einstein special relativity is $E^2 = (mc^2)^2 + p^2c^2$

18. (10%) A diatomic molecule can rotate about its center of mass (**m**$_1$, **m**$_2$) as following figure. Known the angular momentum **L** is quantized.

(A) (2%) Determine the total moment of inertia **I**.

(B) (8%) Determine the energy of a rotating molecule **E**$_J$.



Electron rest mass	$m_e = 9.1095\times10^{-31}$ kg
Positron rest mass	$m_{e+} = 9.1095\times10^{-31}$ kg
Proton rest mass	$m_p = 1.6726\times10^{-27}$ kg
Neutron rest mass	$m_n = 1.6750\times10^{-27}$ kg
Light velocity	$c = 2.998\times10^{8}$ m/s
Electron charge	$e = 1.602\times10^{-19}$ Coul
Planck's constant	$h = 6.626\times10^{-34}$ J·s = 4.1361×10^{-15} eV·s
Permittivity of free space	$\varepsilon_0 = 8.854\times10^{-12}$ F/m