科目：工程數學　　　　　　　　　　　　適用系所：電機工程學系

注意：1.本試題共1頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (10 分) Find the solution of

$$y''-10y'+25y=0$$

2. (10 分) Find the solution of

$$x^2y''-2y=0$$

3. (10 分) Find the solution of this differential equation (DE). You can apply the modified exact method

$$xydx+(2x^2+3y^2-20)dy=0$$

4. (20 分) Newton's law of cooling tells us that, the rate of heat loss of a body is proportional to the temperature difference between the body and the environment. Define T(t) as the temperature of a body with respect to the time, R as the environment temperature, and k as the constant of heat loss. Please use the first-order DE to express the Newton's law of cooling (5 分) and solve this DE (15 分).

5. (20 分) Please derive the Laplace transform of $f(t)=t^2$

6. (30 分) Are the following statements true or false?
A. The basis vectors of a vector space are orthogonal.
B. To span a n-dimensional vector space, we need exactly n vectors.
C. rank(B^TA^T)=rank(AB)
D. For a nxn matrix with nonzero determinant, the number of pivots are n.
E. For nxn matrix with nonzero determinant, its reduced row-echelon form must be an identity matrix I.
F. For a nxn matrix, its row-echelon form has the same determinant