國立臺南大學 109 學年度 **資訊工程學系碩士班** 招生考試 離散數學與線性代數 試題卷

問答題 (配分如各題所示，共 100 分)

1. Let $n \in N$. Prove that $1 + 3 + \cdots + (2n - 1) = n^2$. (10 %)

2. Let R be the relation on the set of real numbers such that aRb if and only if a-b is an integer. Prove that R an equivalence relation. (15 %)

3. Draw the Hasse diagram for the partial ordering $\{(A, B) | A \subseteq B\}$ on the power set $P(S)$ where $S=\{a, b, c\}$. (10 %)

4. The Jacobsthal numbers are defined by the recursion $a_n = a_{n-1} + 2a_{n-2}$ with initial conditions $a_1 = 1$ and $a_2 = 3$. Prove that

$$a_n = round\left\{\frac{2^{n+1}}{3}\right\},$$

   for every nonnegative integer n. Here, $round\{x\}$ denotes the nearest integer to x, rounding up if x is a half-integer. For example, $round\{1.1\}= 1$, $round\{0.92\}=1$ and $round\{1.5\}=2$. (15 %)

5. If A and B are 2×2 matrices and $|A| = -1$ and $|B| = 2$, compute the following determinants. (a)$|3A^2B^{-1}|$ (b)$|(2AB^t)^{-1}|$, where B^t represents the transpose of matrix B. (10 %)

6. In each part, use the information in the table to determine the number of the solutions of the linear system $Ax = b$. That is, will the system have a single, many, or no solutions? (10 %)

	(a)	(b)	(c)	(d)	(e)
Size of A	4×4	3×3	4×3	5×4	5×8
Rank(A)	4	3	2	4	5
Rank[A | b]	4	2	2	5	5

7. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$. Compute A^{-1} (10 %)

8. A linear transformation $T: R^3 \rightarrow R^3$ is defined as $T(x) = Ax$ where vector $x = (x_1, x_2, x_3)$ and matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 4 \end{bmatrix}$. Determine the kernel of the transformation T. (10 %)

9. Show that $\{(1, 2, 3), (-2, 1, 0), (1, 0, 1)\}$ is a basis for R^3. (10 %)