Problem I (15%). Bode's gain-phase relationship states that for any stable minimum-phase transfer function $G(j\omega)$, the phase of $G(j\omega)$ is uniquely related to the magnitude of $G(j\omega)$, by

$$\angle G(j\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{dM}{du} W(u)du \text{ (in radians)},$$

where

$M$ = log magnitude = $\ln |G(j\omega)|$,

$u$ = normalized frequency = $\ln (\omega/\omega_0)$,

$dM/du \approx$ slope $n$, the slope of $G(j\omega)$ in units of decade of amplitude per decade of frequency,

$W(u) = \ln(\coth|u|/2) \approx \pi^2\delta(u)/2$.

Use it to explain why it is desirable to have $n = -1$ for $\omega$ around $\omega_c$, the crossover frequency for about a decade.

Problem II (85%). Suppose the equations of motion for a satellite-attitude control system using a reaction wheel to provide angular motion are:

Satellite: $I\ddot{\phi} = T_c + T_{ex}$

Reaction wheel: $J\dot{r} = -T_c$

Sensor measurement: $\dot{Z} = \dot{\phi} - \alpha Z$

Control: $T_c = -D(s)(Z - Z_d)$

where

$J$ = moment of inertia of the reaction wheel,
$r$ = reaction wheel speed,
$T_c$ = control torque,
$T_{ex}$ = disturbance torque,
$\phi$ = angle to be controlled,
$Z$ = measurement from the sensor,
$Z_d$ = reference angle,
$I$ = satellite inertia (800 kg/m$^2$),
$\alpha$ = sensor constant (1.25 rad/sec),
$D(s)$ = feedback compensator.

(a) (10%) Draw a block diagram for the entire system. Clearly indicate each signal and parameter.
(b) (10%) Suppose $D(s) = K_0$, a real constant. Draw the root locus with respect to $K_0$ for the resulting closed-loop system.
(c) (10%) For what range of $K_0$ is the closed-loop system unstable?
(d) (15%) Now, let $D(s)$ be a lead compensator given by:

$$D(s) = K_1 \frac{s+z}{s+0.8}$$

where both $z$ and $K_1$ are real constants. Where should the zero of the lead compensator be located so that the closed-loop system has a bandwidth $\omega_{BW} \approx 0.05$ rad/sec and a damping ratio $\zeta = 0.5$? Draw the root locus of the compensated system versus $K_1$, and give the value of $K_1$ that allows the specifications to be met.
(e) (10%) For what range of $K_1$ is the system unstable?
(f) (10%) What is the steady-state error (the difference between $Z$ and some reference input $Z_d$) to a constant disturbance torque $T_{ex}$ for the design of part (d)?
(g) (10%) What is the type of this system with respect to rejection of $T_{ex}$?
(h) (10%) Draw the Bode plot of the open-loop system, combined with the lead compensator designed in part (d). Clearly indicate the asymptotes, corner frequencies, slopes, gain margin, and phase margin.

試題隨卷繳回