第一部份 (填充題，90 分)

問題 1 至 6 中總共有 21 個空格($A1, A2, \ldots, A5, B1, B2, C1, C2, C3, C4, D1, D2, \ldots, D5, E1, E2, F1, F2, F3$)。請您根據題意，將適當的數字、向量、邏輯值、函數、符號或文字等作答於答案卷。例如：$A1 = \tan(5t)$, $A2 = 27$ 等。

1.(25%)Many different physical systems can be discribed by a linear second-order differential equation similar to the differential equation of forced motion with dampling: $m\frac{d^2x(t)}{dt^2} + b\frac{dx(t)}{dt} + kx(t) = f(t)$. If $i(t)$ denotes current in a standard inductor-resister-capacitor (LRC) series electrical circuit, then, by Kirchhoff's second law, the sum of the voltage drops across the inductor, resister, and capacitor equals the input voltage $E(t)$ impressed on the circuit: that is, $L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}q(t) = E(t)$. But the charge $q(t)$ on the capacitor is related to the current $i(t)$ by $i(t) = \frac{dq(t)}{dt}$, so the above-mentioned equation becomes the linear second-order differential equation: $L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t) = E(t)$. If $E(t) = 0$, the electrical vibrations of the circuit are said to be free. Furthermore, if $L = 0.25$ henry(h), $R = 10$ ohms(Ω), $C = 0.001$ farad(f), $q(0) = q_0$ coulombs(C), and $i(0) = 0$, find the charge on the capacitor in the LRC series circuit as $q(t) = q_0 \boxed{(A1)} \left(\cos(60t) + \boxed{(A2)} \sin(60t)\right)$. Also, by computing the value of $R^2 - 4L/C$, it can be shown that the circuit is $\boxed{(A3)}$ -damped. Finally, if $E(t) = E_0 \sin(100t)$, find the steady-state charge $q(t)$, denoted as $q_{ss}(t) = \boxed{(A4)} \sin(100t) + \boxed{(A5)} \cos(100t)$

$\boxed{(A1)} = ?$ (5 points); $\boxed{(A2)} = ?$ (5 points); $\boxed{(A3)} = ?$ (5 points); $\boxed{(A4)} = ?$ (5 points); $\boxed{(A5)} = ?$ (5 points)

2.(9%). The solution for the following Dirichlet problem

$$\nabla^2 u = u_{xx} + u_{yy} = 0, \quad (-\infty < x < \infty, 0 < y < \infty)$$

$$u(x,0) = \begin{cases} 0, & -\infty < x < 0 \\ 20, & 0 \le x < \infty \end{cases}$$

can be written as $u(x,y) = 10 - A\tan^{-1}(B)$.

$\boxed{(B1)} = A = ?$ (5 points), $\boxed{(B2)} = B = ?$ (4 points)

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3. (16%)If the particular solution that satisfies the following equation and initial conditions:

$$xy'' + 3y' + 25xy = 0, \quad 0 < x < \infty; \quad y(0) = 12, \quad y'(0) = 0$$

can be written as $y(x) = Ax^B J_1(Cx) + Dx^B Y_1(Cx)$

$\boxed{(C1)} = A = ?$ (5 points), $\boxed{(C2)} = B = ?$ (3 points)

$\boxed{(C3)} = C = ?$ (3 points), $\boxed{(C4)} = D = ?$ (5 points)

Note that

$$J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\Gamma(2+n)} \left(\frac{x}{2}\right)^{2n+1} \approx \frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{384} + \cdots$$

$$Y_1(x) = \frac{\cos\pi J_1(x) - J_{-1}(x)}{\sin\pi} \approx \frac{1}{\pi}\left[\left(\gamma + \ln\left|\frac{x}{2}\right|\right) J_1(x) - \frac{2}{x} - \frac{x}{2}\left(1 - \frac{3}{16}x^2 + \frac{5}{288}x^4 + \cdots\right)\right]$$

$\gamma \approx 0.57721556$

4.(15%)Given a matrix C satisfying $v^T C v \geq 0$ for vector v in $\mathbb{R}^n$, answer **true** or **false** to the following statements.

$\boxed{(D1)}$ $C = C^T$.

$\boxed{(D2)}$ $C^T = C^{-1}$.

$\boxed{(D3)}$ $v^T C C^T v \geq 0$.

$\boxed{(D4)}$ The eigenvalues of C are nonnegative.

$\boxed{D(5)}$ The columns of C form a basis for a nonzero subspace of $\mathbb{R}^n$.

$\boxed{(D1)}$=? (3 points) $\boxed{(D2)}$=? (3 points) $\boxed{(D3)}$=? (3 points) $\boxed{(D4)}$=? (3 points) $\boxed{D(5)}$=? (3 points)

5.(10%) Given n data points $(a_1, b_1), (a_2, b_2), \ldots, (a_n, b_n)$ on the two-dimensional (x, y)-plane with $\sum_{i=1}^{n} a_i = n$ and $\sum_{i=1}^{n} b_i = 2n$, find the equation of the least-squares line $y = \boxed{(E1)} + \boxed{(E2)}(x-1)$.

$\boxed{(E1)}$=? (5 points) $\boxed{(E2)}$=? (5 points)

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6.(15%)(a) Let $A=\begin{bmatrix}1 & 2 & 1\\ -2 & 1 & 1\\ 1 & -1 & 1\end{bmatrix}$, $b=\begin{bmatrix}2\\ -1\\ 2\end{bmatrix}$ and $Ax=b$. $\boxed{(F1)}=x=?$ (5 points)

(b) Let $A=\begin{bmatrix}1 & 1\\ -1 & 1\\ 1 & 1\end{bmatrix}$ $b=\begin{bmatrix}2\\ 1\\ 0\end{bmatrix}$. Then there exists x, such that $\|Ax-b\|\le\|Az-b\|$ for every $z\in\mathbb{R}^2$.

$\boxed{(F2)}=x=?$ (5 points)

(c) Let $A=\begin{bmatrix}1 & -1 & 0\\ 1 & 1 & 1\end{bmatrix}$, $b=\begin{bmatrix}4\\ 3\end{bmatrix}$. Then there exists x, such that $\|x\|\le\|z\|$, in which $Ax=b$ and all $z\in\mathbb{R}^3$ satisfying $Az=b$. $\boxed{(F3)}=x=?$ (5 points)

第二部份 (證明題，10 分)

7.(10%) If p and q are eigenvectors of a real symmetric matrix that correspond to distinct eigenvalus, prove that p and q are orthogonal.

試題隨卷繳回