

國立聯合大學 109 學年度碩士班考試招生

電子工程學系碩士班入學考試試題

科目： 第 1 頁共 1 頁

1. Please solve the following 1st order differential equations.

a. $3y' = 4xe^{x^2} / y^3$ (5%)

b. $y' - \frac{3}{x}y = 2x^2$ (5%)

2. Please solve the following 2nd-order differential equations.

a. $y'' + 2y' + y = 9e^{-x}$ (10%)

b. $x^2y'' - 4xy' + 6y = \ln(x^2)$ (10%)

3. Please sketch the following functions and then find their convolution integral $f(t) * g(t)$,

$$f(t) = \begin{cases} 1, & 0 \leq t < 2 \\ -1, & 2 \leq t < 4 \\ 0, & \text{elsewhere} \end{cases}, \quad g(t) = \begin{cases} 1, & 0 \leq t < 4 \\ 0, & \text{elsewhere} \end{cases}. \quad (10\%)$$

4. Use the Laplace transform to solve the integral equation $f(t) = e^{-(t-t_0)}u(t-t_0) + \int_0^t f(t-\tau)d\tau$. (10%)

5. Use the Laplace transform to solve the differential equation $y'' + 5y' + 4y = u(t) - u(t-5)$, $y(0) = 1$, $y'(0) = 0$. (10%)

6. Let $f(x) = \begin{cases} -k, & -\pi \leq x < 0 \\ k, & 0 \leq x < \pi \end{cases}$, please solve the following questions.

a. Sketch $f(x)$ carefully. (3%)

b. Find the Fourier series of $f(x)$. (9%)

c. Use the result of question 6b to show that $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n-1} = \frac{\pi}{4}$. (3%)

7. Let $f(t) = e^{-|t|}$, $-\infty < t < \infty$, please solve the following questions.

a. Sketch $f(t)$ carefully. (3%)

b. Find its Fourier transform. (6%)

c. Find the amplitude and the phase from the result of 7b. (6%)

8. Consider three vectors $\mathbf{a} = 2\vec{i} - 2\vec{j} + 4\vec{k}$, $\mathbf{b} = 8\vec{j} + 4\vec{k}$ and $\mathbf{c} = -5\vec{i} - \vec{j} + 2\vec{k}$,

please solve the following questions.

a. Show that these three vectors are orthogonal. (5%)

b. Express the vector $\mathbf{x} = -22\vec{i} + 6\vec{j} + 8\vec{k}$ in terms of \mathbf{a} , \mathbf{b} and \mathbf{c} . (5%)