注意：考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。

國立清華大學 109 學年度碩士班考試入學試題

系所班組別：聯合招生

科目代碼：9803

考試科目：電磁學

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 109 學年度碩士班考試入學試題

系所班組別：聯合招生 (0598)

考試科目（代碼）：電磁學 (9803)

　*請在【答案卷】作答

電磁常數：

permittivity	$\varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m
permeability	$\mu_0 = 4\pi \times 10^{-7}$ H/m
light speed	$c = 3 \times 10^8$ m/s

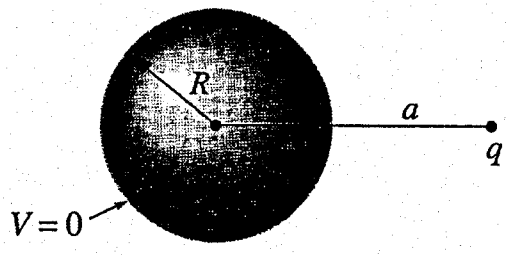
注意事項：請以 SI 制單位回答下面所有問題

1. (15%) Consider a point charge q situated a distance a from the center of a grounded conducting sphere of radius R $(a > R)$. This electrostatic problem can be solved by the method of images.

(a) Find the charge q' of the mirror image and the distance b of the image to the center of the sphere.

(b) Find the induced surface charge distribution $\sigma(\theta)$ on the sphere.

(c) Calculate the force acting on the charge q.

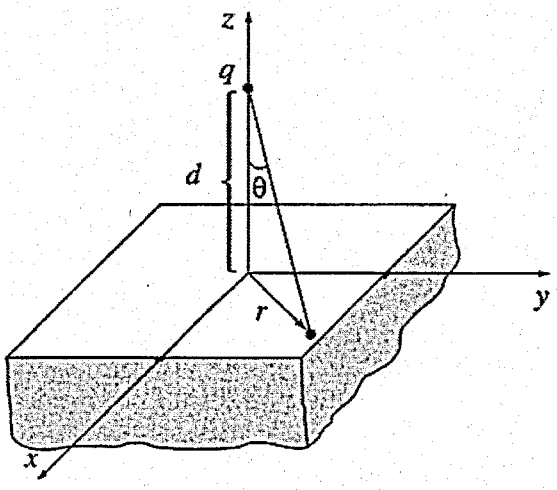


2. (15%) Suppose the entire region below the plane $z = 0$ is filled with uniform dielectric material of susceptibility χ_e. A point charge q is placed at a distance d above the region.

(a) Find the bound charge distribution σ_b on the surface

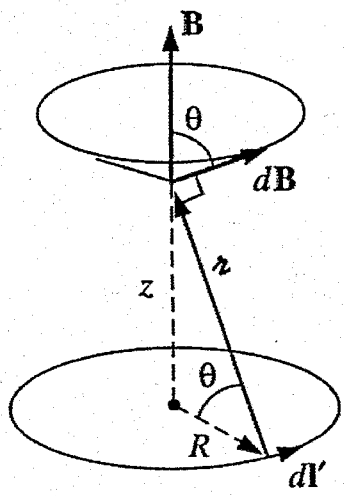
(b) Calculate the total bound charge q_b on the surface.

(c) Calculate the force acting on the charge q.

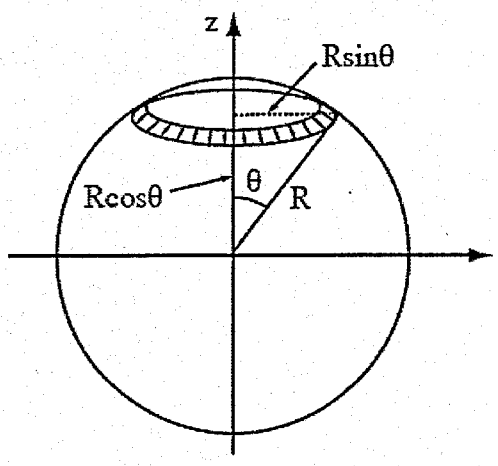


3. (15%)

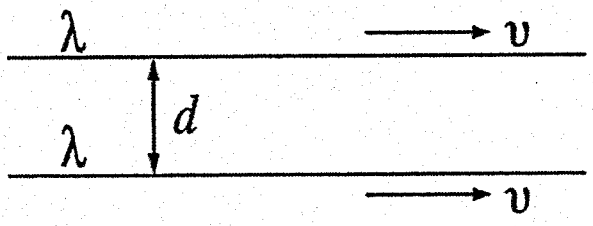
(a) Find the magnetic field **B** a distance z above the center of a circular loop of radius r, which carries a steady current I.



(b) Calculate the magnetic field **B** at the center of a uniformly charged spherical shell of radius R and total charge Q, spinning at a constant angular velocity ω.



(c) Two infinite straight line charge λ, a distance d apart, move along at a constant speed v. How great would v have to be in order for the magnetic attraction to balance the electrical repulsion?



4. (15%)

(a) A long cylinder of radius R carries a magnetization $\mathbf{M} = 2r^2\hat{\phi}$ where r is the distance from the axis and $\hat{\phi}$ is the azimuthal unit vector. Find the magnetic field $\mathbf{B}$ due to $\mathbf{M}$ for points inside and outside the cylinder.

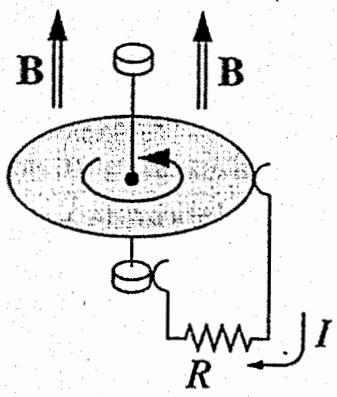
Formula: Curl in cylindrical coordinates: $\nabla \times \mathbf{v} = \left[\frac{1}{r}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{r}} + \left[\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right]\hat{\boldsymbol{\phi}} + \frac{1}{r}\left[\frac{\partial (r v_\phi)}{\partial r} - \frac{\partial v_r}{\partial \phi}\right]\hat{\mathbf{z}}$

(b) A long copper rod of radius R carries a uniformly distributed (free) current I. Find $\mathbf{H}$ inside and outside the rod.

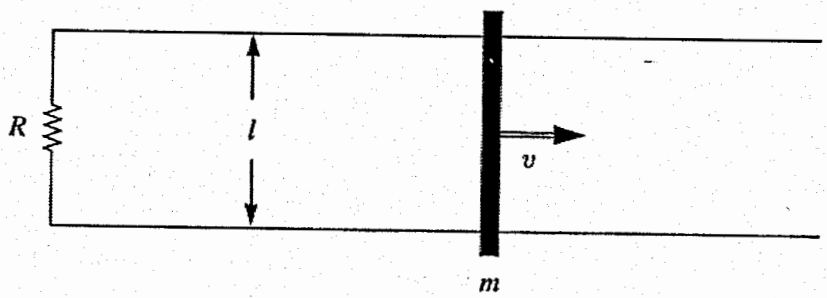
(c) An infinite solenoid (n turns per unit length, current I) is filled with linear material of susceptibility χ_m. Find the magnetic field $\mathbf{B}$ inside and outside the solenoid.

5. (15%)

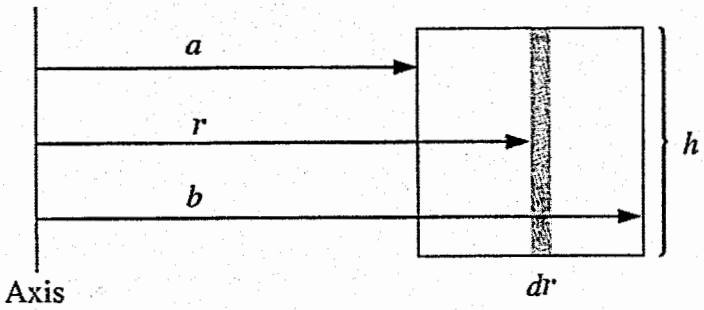
(a) A metal disk of radius a rotates with angular velocity ω about a vertical axis, through a uniform magnetic field $\mathbf{B}$, pointing up. A circuit is made by connecting one end of a resistor to the axle and the other end to a sliding contact, which touches the outer edge of the disk. Find the current in the resistor.



(b) A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance ℓ apart. A resistor R is connected across the rails and a uniform magnetic field $\mathbf{B}$, pointing into the page, fills the entire region. If the bar starts out with speed v_0 at $t = 0$, and is left to slide. Find the displacing distance as the bar stops sliding.



(c) Find the self-inductance of a toroidal coil with rectangular cross section (inner radius a, outer radius b, height h), that carries a total of N turns.



6. (15%) The wave equations for $\mathbf{E}$ and $\mathbf{B}$ in conductors can be written as $\nabla^2\mathbf{E} = \mu\epsilon\frac{\partial^2}{\partial t^2}\mathbf{E} + \mu\sigma\frac{\partial}{\partial t}\mathbf{E}$, $\nabla^2\mathbf{B} = \mu\epsilon\frac{\partial^2}{\partial t^2}\mathbf{B} + \mu\sigma\frac{\partial}{\partial t}\mathbf{B}$.

(a) Find the complex wave number $\tilde{k} = k_1 + ik_2$ in terms of ϵ, μ, σ, and ω for complex plane-wave solutions for $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$.

(b) Find the skin depth δ of the electromagnetic wave in poor ($\sigma \ll \omega\epsilon$) and good ($\sigma \gg \omega\epsilon$) conductors, respectively. Does the skin depth depend on the angular frequency ω in the two cases?

(c) Show that in a conductor, the time-averaged magnetic energy density $\langle u_B \rangle$ of an electromagnetic plane wave is always larger than the time-averaged electric energy density $\langle u_E \rangle$.

7. (10%) Consider a rectangular wave guide with dimension $2.5\text{cm} \times 1.5\text{cm}$.

(a) What TE modes will propagate in this wave guide if the driving frequency is $1.5 \times 10^{10}\text{Hz}$.

(b) Suppose you wanted to excite only one TE mode. What range of frequencies should you use? What are the corresponding wavelengths (in open space)?