注意：考試開始鈴響前，不得翻閱試題，

並不得書寫、畫記、作答。

國立清華大學 109 學年度碩士班考試入學試題

系所班組別：資訊工程學系

科 目 代 碼 ：2301

考 試 科 目 ：基礎計算機科學

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「⬇由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 109 學年度碩士班考試入學試題

系所班組別：基礎計算機科學

考試科目（代碼）：2301

1. (10 points) Consider $L = \{ a^n b^n \}$ and the statement

   $S = \exists$ *integer m* ($\forall$ string $w \in L, |w| \geq m$ ($\exists x y z = w$ ($\forall n$ ($x y^n z \in L$)))).

   Write the statement of $\neg S$ (the negation of S).

   Hint: $\neg S =$ (_ integer m ($\exists$ string $w \in L, |w| \geq m$ (__ $x y z = w$ (__ n (___))))

2. (5 points) (a) What is a spanning tree?

   (5 points) (b) Given an undirected, weighted graph in Figure 1, what is the minimum spanning tree (MST)?

   (5 points) (c) Describe the sequence of adding edges to form the MST of the graph in Figure 1 using the greedy Kruskal's algorithm.

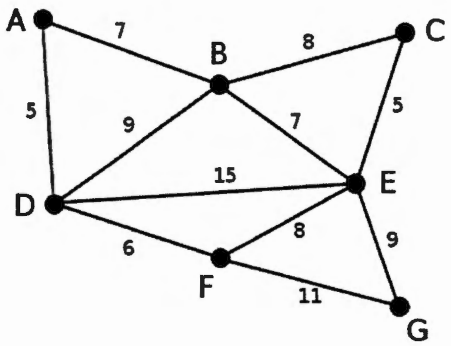


Figure 1 A weighted Graph

   Hint: (1) AD (2) ____ (3) ____ (4) ____ (5) ____ (6) ____

3. (8 points) Use the Euclidean algorithm to find the greatest common divisor of 167,076 and 1,928,737.

4. (5 points) Five people occupy five seats. If five seats are arranged in a circle, how many different ways can the five people select their seats?

5. (4 points) Let $G = (V, E)$ be a graph. If V has twelve members, in which four members each has a degree of three, and the degree of each remaining member is five, how many members does E have?

6. (8 points) A class at a college consists of 19 students who sit at a circular table. The instructor wants each student to sit next to two different classmates each day. For how many days can they do this?

7. (2 x 5 points) Please find the tight asymptotic upper bounds of the following recurrences in big-O notation and also justify your answers.

   (a) $T(n) = T(n\text{-}1) + 2n$
   (b) $T(n) = 2T(n\text{-}1) + n$

8. (8 points) Given a sequence of n integers $A = (a_1, a_2, ..., a_n)$, the *longest increasing subsequence problem* is to find a longest subsequence $(a_{i_1}, a_{i_2}, ..., a_{i_k})$ of A such that $i_1 < i_2 < \cdots < i_k$ and $a_{i_1} < a_{i_2} < \cdots < a_{i_k}$. For example, $(1,2,5,8)$ is a longest increasing subsequence of $(4,1,7,5,2,5,8,4)$. Please use the dynamic programming technique to design an $O(n^2)$ time algorithm for solving the longest increasing subsequence problem. Please also justify your algorithm and its time complexity.

9. (7 points) Given a set S of n numbers, the *k-partition problem* is to determine whether or not S can be partitioned into k subsets of the same sum. For example, let $S = \{1,2,9,12,18\}$. Then for the two-partition problem, we indeed can partition S into two subsets $S_1 = \{1,2,18\}$ and $S_2 = \{9,12\}$ such that the sum of all elements in S_1 equals to the sum of all elements in S_2. In fact, it can be proved that the two-partition problem is NP-complete. In the situation where the two-partition problem is already NP-complete, please prove that the three-partition problem is also NP-complete.

10. (4 x 2 points) **True or False Questions.** If your answer is False, please briefly justify. (No point is given without justification if the answer is False)

    (a) If f(n) = O(g(n)), we can say that g(n) ≥ f(n) for n > 1.

    (b) Merge Sort has worst-case time complexity O(n log n), while the worst-case time complexity of Insertion Sort is O(n^2). One weakness of Merge Sort is that it requires additional space. Therefore, if space allows, we should always use Merge Sort for better efficiency.

    (c) Searching a specific key in a binary search tree takes O(log n) time, where n is the number of keys in the binary search tree.

(d) Given the pre-order and level-order traversal sequences, we can construct a unique binary tree.

11. (2x4 points) Given the AVL tree below, please answer the following sub-problems.

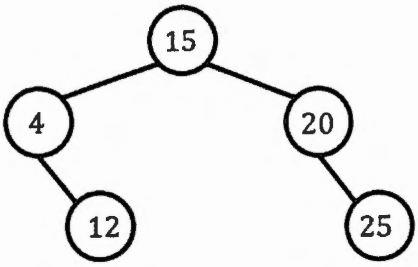
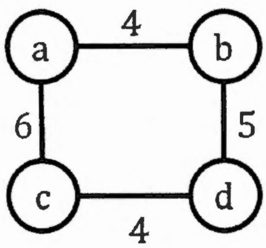


Figure 2. The given AVL tree.

(a) Please sequentially insert the following keys into the given AVL tree: 17, 19, 18. Please show the final result of the AVL tree after all the keys are inserted. Only the final result is needed, no step-by-step illustration is required.

(b) Continue with the previous sub-problem. After the keys in sub-problem (a) are inserted, please sequentially delete keys 25 and 17 (when deleting a non-leaf node from the AVL tree, please replace it by the node with the largest key in its left subtree). Please show the final AVL tree only (no step-by-step illustration is required).

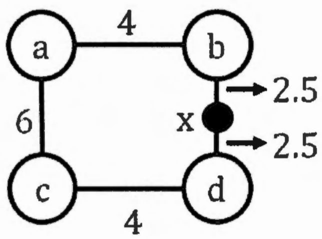
12. Given a connected and weighted graph $G = (V, E)$, where all the edge weights are positive integers. The eccentricity $\epsilon(v)$ of a vertex $v \in V$ is the greatest shortest path distance between v and any other vertex. That is, $\epsilon(v) = max_{u \in V} d(v, u)$, where $d(v, u)$ denotes the shortest path distance between vertices v and u. For example, in the following figure, $\epsilon(b) = \max\{d(b, a\}, d(b, c), d(b, d)\} = 9$.



Please answer the following questions.

(a) (3 points) A **center** of a graph is a vertex that incurs the minimum eccentricity. That is, a center c is defined as: $c = \operatorname{argmin}_{v \in V} \epsilon(v)$. Is it possible for a graph to have more than one center? If yes, please provide an example; If no, please provide a proof.

(b) (2 x 3 points) In contrast to the center, which must be a vertex, an **absolute center** is a point that can be on an edge or on a vertex, such that its maximum shortest path distance to all vertices is minimum. Take the following graph as an example, the point x is an absolute center, because the shortest path distances from x to vertices a, b, c, d are 6.5, 2.5, 6.5, 2.5, respectively. That is, the maximum shortest path distance from x to all the other vertices is 6.5, which is minimum among all possible cases.



Given the definition of the absolute center, we know that there may be multiple absolute centers in a graph. So, please answer the following questions.

(b-i) If there are multiple absolute centers in a graph, can all of them be on vertices, i.e., no absolute center is on an edge? If yes, please provide an example; If no, please provide a proof.

(b-ii) If there are multiple absolute centers in a graph, can some of them be on vertices, and some of them be on edges at the same time? If yes, please provide an example; If no, please provide a proof.