

國立中正大學

109 學年度碩士班招生考試

試題

[第 1 節]

科目名稱	數學
系所組別	資訊工程學系- 甲組 乙組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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1. (10%) Compute the determinant of

$$A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$$

2. Find the dimensions of the following subspaces of R^4 .

(a) (5%) All vectors of the form $(a, b, c, 0)$.

(b) (5%) All vectors of the form (a, b, c, d) , where $d = a + b$ and $c = a - b$.

(c) (5%) All vectors of the form (a, b, c, d) , where $a = b = c = d$.

3. (10%) The matrix $A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$ has eigenvalues 1, -2 and -2. Find an orthogonal matrix P and a diagonal matrix D such that $A = PDP^T$.

4. (6%) Let \mathbf{v}_1 and \mathbf{v}_2 denote the following vectors in R^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 2/3 \\ -1/3 \\ -2/3 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} -\sqrt{2}/2 \\ 0 \\ -\sqrt{2}/2 \end{bmatrix}$$

Find a vector \mathbf{v}_3 so that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ form an orthonormal basis for R^3 . How many choices are there for the answer?

5. Let A be the 2x2 matrix with eigenvalues $\lambda_1 = 2$, and $\lambda_2 = -1$ for which $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ are corresponding eigenvectors .

(a) (3%) Find A .

(b) (2%) What are the eigenvalues of $(A+I)$?

(c) (4%) Calculate $(A+I)^{100}$.

6. (5%) An n -dimensional hypercube, or n -cube, denoted by Q_n , is a graph that has vertices representing the 2^n bit strings of length n . Two vertices are adjacent if and only if the bit strings that they represent differ in exactly one bit position. By the definition of Q_n , draw the graph Q_4 .

7. (10%) Solve the recurrence relation $a_n^2 - 2a_{n-1}^2 = 1$ for $n \geq 1$ where $a_0 = 1$.

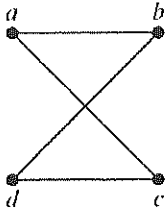
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8. (10%) How many paths of length four are there from a to d in the following graph? List all paths.



9. If a and b are integers and m is a positive integer, then a is congruent to b modulo m if m divides $a - b$. We use the notation $a \equiv b \pmod{m}$ to indicate that a is congruent to b modulo m .
- (a) (5%) Find an inverse of 144 modulo 233.
- (b) (5%) Solve the congruence $144x \equiv 7 \pmod{233}$
- 10.
- (a) (3%) How many cards must be selected from a standard deck of 52 cards to guarantee that at least five cards of the same suit are chosen?
- (b) (2%) How many must cards be selected to guarantee that at least five hearts are selected?
11. A string that contains only 0s and 1s is called a binary string.
- (a) (5%) Find a recurrence relation for the number of binary strings of length n that contain three consecutive 0s.
- (b) (2%) What are the initial conditions?
- (c) (3%) How many binary strings of length seven do contain three consecutive 0s?