國立政治大學 109學年度 碩士班 招生考試試題

考試科目	統計學B	所別	金融學系財務工程與金融科技組一般生	考試時間	2月 7日(五)第三節

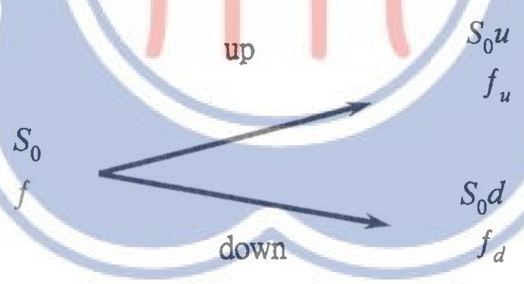
1. (15%) Statistics and Econometrics

Please describe one assignment or one project you have done for financial analysis, where the assignment or project needs the estimating methods or the testing methods. Please give the name and the procedure and property of the estimation or the testing methods in detail. (15%)

2. (25%) Binomial model and no-arbitrage argument

Based on a binomial model and no-arbitrage argument, we can generalize the no-arbitrage argument just presented by considering a stock whose price is S_0 and an option on the stock shoes current price is f. We assume that option fasts for time and that during the life of the option the stock price can tither move up from S_0 to a new level, S_0u, where $u>1$, or down from S_0 to a new level, S_0d, where $d<1$, and $u\,d=1$.

If the stock price moves up to S_0u, we suppose that the payoff from the option is f_u; if the stock price moves down to S_0d, we suppose that the payoff from the option is f_d.



1. We imagine a portfolio consisting of a long position in △ shares and a short position in one option. Please Calculate the value of △ that makes the portfolio riskless. (5%)
2. What is the value of the option f at the one-step binomial tree under the no-arbitrage argument? (5%)
3. What is the value of the option f at the n-step binomial tree under the no-arbitrage argument? (5%)
4. What is the value of the option f at the continuous time under the no-arbitrage argument when the n-step binomial tree goes to infinity? (10%)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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3. **(60%) Black-Scholes Option Pricing Formula**

Consider a Brownian motion process $\{B(t), t \geq 0\}$. $B(0)=0$ and $B(t)$ is normal with mean 0 and variance t, where its density function is given by

$$f_t(b) = \frac{1}{\sqrt{2\pi t}} e^{-b^2/2t},$$

and the process $B(t)$ has stationary and independent increments, where $B(t_1)$, $B(t_2)-B(t_1)$, …, $B(t_n)-B(t_{n-1})$ for $t_1 < \ldots < t_n$ are independent and $B(t_k)-B(t_{k-1})$ is normal with mean 0 and variance $t_k - t_{k-1}$. Assume $B(t_1)=b_1$, $B(t_2)=b_2$, …, $B(t_n)=b_n$.

(a). Please give the joint density of $B(t_1)$, $B(t_2)$, …, $B(t_n)$. (5%)

(b). Please find the covariance of $B(t)$ and $B(s)$, $Cov(B(t), B(s))$ where $s < t$. (5%)

(c). Please find the conditional distribution of $B(s)$ given $B(t) = C$ where $s < t$. (5%)

Now, let the dynamics of the stock price be $S(T) = S(0)\exp\{(r - 0.5\sigma^2)T + \sigma B(T)\}$ under the risk neutral measure at time T, where $S(0)$ denotes the stock price at time 0, r is the riskless rate, σ is the volatility of the log stock price under risk-neutral probability measure.

(d). Please find the mean and variance of $S(T)$ under the risk-neutral probability measure. (10%)

(e). If the underlying asset of the option is the stock, what is the theoretical value of the stock option with strike price K and maturity T at time 0 under the risk-neutral probability measure? (Hint: To derive Black-Scholes Option Pricing Formula.) (10%)

(f). Please find the estimators of μ and σ by the maximum likelihood estimation (MLE) at the physical (real) probability measure based on the stock prices data S_i i=1,2,…, n for n days. (10%)

(g). What is the implied volatility? (5%)

(h). Please find the implied volatility based on the option price C. (10%)?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。