國立臺北科技大學109學年度碩士班招生考試

系所組別：1120 機械工程系機電整合碩士班乙組

第一節 工程數學 試題

注意事項：
1. 本試題共5題，每題20分，共100分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. The general first-order linear differential equation is given as

$$\frac{dy}{dx} + p(x)y = q(x). \tag{1}$$

By treating Eq. (1) as **an exact equation**,

   (a) (10 points) find the integrating factor of Eq. (1) with **a differential form**, and then

   (b) (10 points) derive the general solution of Eq. (1).

2. (20 points) By using the convolution theory and the Laplace transform, find the $g(t)$ of following equation

$$\int_0^t \int_0^\tau g(t-\tau)g(\tau-k)g(k)d\tau dk = \frac{t^5 \exp(3t)}{120}.$$

3. (a) (10 points) Calculate line integral shown in Fig. 1 using $\mathbf{F} = (xy^3)\,\mathbf{i} + (x^2 - y^2)\mathbf{j}$.

   (b) (10 points) Verify Green's lemma for the case using $\mathbf{F} = (xy^3)\,\mathbf{i} + (x^2 - y^2)\mathbf{j}$ with enclosed area $\mathcal{S}$ shown in Fig. 1.
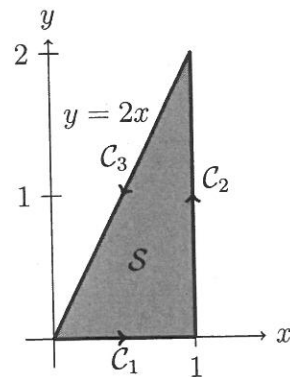

Figure 1: Enclosed area.

4. (a) (10 points) Find the Fourier series of the function $f(x) = x + \pi$ on the given interval $-\pi < x < \pi$.

   (b) (10 points) Use the result of 4. (a) to show $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$.

5. (20 points) On the complex plane, evaluate $\oint_C \frac{dz}{z^2 + 2}$ along the closed circular contour C with a radius of 4 which is centered at origin, in the counterclockwise direction. Here $z = x + iy$.