

科目：工程數學(線性代數+微分方程)

編號：352 適用：電機系

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題
共 / 頁
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1. (15%) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, and assume that the determinant of A , $\det(A) = 5$. Find

(a) (5%) $\det(A^{-1})$.

(b) (5%) $\det(-2A^T)$.

(c) (5%) $\det\left(\begin{bmatrix} a & g & d \\ 2b & 2h & 2e \\ -c & -i & -f \end{bmatrix}\right)$.

2. (10%) Let $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix}$.

(a) (5%) Find an orthonormal basis of the null space of matrix A .

(b) (5%) Find an orthonormal basis of the column space of matrix A .

3. (10%) Let A be an invertible matrix and \mathbf{x} is an eigenvector of A with associated eigenvalue 3.

(a) (5%) Find the eigenvalue of $A - 4I$ associated with \mathbf{x} , where I is an identity matrix.

(b) (5%) Find the eigenvalue of $-2A^{-1}$ associated with \mathbf{x} .

4. (15%) True or false.

(a) (4%) An invertible matrix is always diagonalizable.

(b) (4%) Two similar matrices share the same eigenvectors and eigenvalues.

(c) (4%) The columns of an orthogonal matrix are always linearly independent.

(d) (3%) If A is a 3×3 matrix, then $\det(2A) = 6\det(A)$.

5. (20%) Solve the following initial-value problems.

(a) (6%) $y'' + 9y = 15e^x$ with $y(0) = 2$ and $y'(0) = -4$.

(b) (7%) $x^2y'' + 11xy' + 25y = 0$ with $y(1) = 1$, and $y'(1) = -1$.

(c) (7%) $y''' + 3y'' + 3y' + y = 8\sin(x)$ with $y(0) = -1$, $y'(0) = -3$ and $y''(0) = 5$.

6. (20%) Find the inverse Laplace transform of each $X(s)$ below, denoted by $x(t)$.

(a) (6%) $X(s) = \frac{s}{s^2 - 2s - 8}$.

(b) (6%) $X(s) = \frac{2s-10}{s^3} e^{-2s}$

(c) (8%) $X(s) = \ln\left(\frac{s^2+1}{(s-1)^2}\right)$.

7. (10%) Solve the following problems by the Laplace transform.

(a) (5%) $y'' + 9y = 4\delta(t-2)$ with $y(0) = -1$, and $y'(0) = 0$, where $\delta(t)$ is the impulse function.

(b) (5%) $y'' - 5y' + 6y = 6u(t-1)$ with $y(0) = 0$, and $y'(0) = 0$, where $u(t)$ is the unit-step function.