國立中正大學
109 學年度碩士班招生考試
試 題
[第 1 節]

科目名稱	機率
系所組別	通訊工程學系-通訊丙組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1. (10%) Let the random variable Y be defined by $Y = X^2$, where X is a Gaussian random variable with expected value $\mathrm{E}\{X\} = 1$ and variance $\mathrm{VAR}\{X\} = 4$. Find the probability density function (pdf) of Y.

2. (10%) We have two coins, one is fair and thus lands heads up with probability $1/2$ and one is unfair and lands heads up with probability p, $p > 1/2$. One of the coins is selected randomly and tossed n times yielding n straight tails. What is the probability that the unfair coin was selected given the the above observation?

3. (10%) Let X_1, X_2, X_3 are independent, exponentially distributed random variables with parameter λ_1, λ_2, and λ_3 respectively. Find the characteristic function of Z, where $Z = X_1 + X_2 + X_3$.

4. (20%) Let X and Y have joint pdf

$$f_{XY}(x,y) = k(x+y), \quad \text{for } 0 \le x \le 1, 0 \le y \le 1.$$

   (a) (10%) Find k.

   (b) (10%) Find the probability $P(X < Y)$.

5. (10%) Find $\mathrm{E}\{X^2 e^Y\}$ where X and Y are independent random variables, X is a zero-mean, unit-variance Gaussian random variable, and Y is a continuous uniform random variable in the interval $[0, 3]$.

6. (10%) Let $Z = X/Y$. Find the probability density function of Z if X and Y are independent and both exponentially distributed with mean one.

7. (10%) The number N of packet arrivals in t seconds at a multiplexer is a Poisson random variable with $\alpha = \lambda t$, where λ is the average arrival rate in packets/second. Let Z be the time until the first packet arrival. Find the probability of the event "$Z > t$".

8. (20%) The probability density function (pdf) of a Chi-square random variable, X with $2n$ degrees of freedom is given by

$$f_X(x) = \begin{cases} \frac{1}{(n-1)!} x^{n-1} e^{-x} & , x \ge 0 \\ 0 & , \text{otherwise} \end{cases}$$

   where n is a positive integer.

   (a) (10%) Find $\mathrm{E}\{e^{-X/4}\}$. Hint: Use the fact that $\int_0^\infty t^{n-1} e^{-t} dt = (n-1)!$ for any positive integer n

   (b) (10%) Let Y be a Chi-square random variable with 2 degrees of freedom. Y is independent of X. Find the probability $P\left(Y \le \frac{X}{4}\right)$.