國立中正大學
109 學年度碩士班招生考試
試 題

[第 2 節]

科目名稱	機率
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。

2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。

3. 入場後於考試開始 40 分鐘內不得離場。

4. 全部答題均須在試卷（答案卷）作答區內完成。

5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。

6. 試題須隨試卷繳還。

Probability

1. (40%) Let X be a random variable with the following probability density function:

$$f_X(x) = ce^{-\alpha|x|}, \ -\infty < x < \infty.$$

   (a) (10%) Find the constant c.

   (b) (10%) Find the probability $P(|X| > v)$ for some positive number v.

   (c) (10%) Let $Y = 2X + 1$. Find the probability density function of Y.

   (d) (10%) Let $Y = 2X + 1$. Find the cumulative distribution function of Y.

2. (10%) Consider the following joint cumulative distribution function (CDF):

$$F_{X,Y}(x, y) = 1 / \left(1 + e^{-2x} + 3e^{-3y}\right), \ -\infty < x < \infty, \ -\infty < y < \infty.$$

   Find the marginal probability density function for X.

3. (20%) Consider the following joint probability density function:

$$f_{X,Y}(x, y) = \begin{cases} 2 & \text{if } 0 \le x \le y \le 1. \\ 0 & \text{otherwise.} \end{cases}$$

   (a) (10%) Find the marginal probability density function (pdf) of X.

   (b) (10%) Find the covariance of X and Y.

4. (10%) Two cards are randomly chosen without replacement from an ordinary deck of 52 cards. Let B be the event that both cards are aces; let A be the event that at least one ace is chosen. Find $P(B|A)$.

5. (10%) Let X and Y be two independent Gaussian random variables with the following statistics: $E\{X\} = 1$, $VAR\{X\} = 4$, $E\{Y\} = 3$, and $VAR\{Y\} = 9$. Let $Z = X - Y$. Find the probability of the event "$Z > 10$" and express it using the Q-function.

6. (10%) Balls numbered 1 through 20 are placed in a bag. Three balls are drawn out of the bag without replacement. What is the probability that all the balls have odd numbers on them?