

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Consider the inverted pendulum system shown in Figure 1. Assume that the mass of the mobile base is m_1 and the inverted pendulum is m_2 with a massless rod of the length l . The angle of the rod to the vertical axis is θ . For the system, $y(t)$, the horizontal movement, is considered as the output with the input $u(t)$ as the driving force on the mobile base.

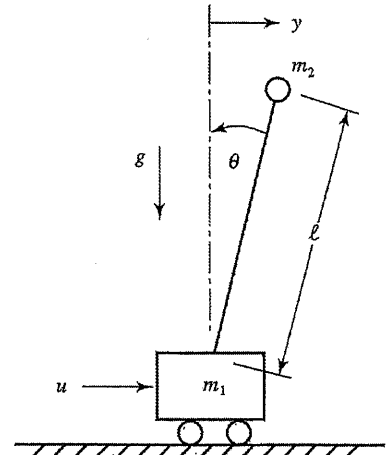


Figure 1. Inverted pendulum system.

- (1) Derive the dynamic model of the inverted pendulum system.
- (2) Find out the transfer function of the inverted pendulum system with $y(t)$ as the output and $u(t)$ as the input if θ is large.
- (3) If $\theta(t)$ is very small, find out the transfer of the system with $\theta(t)$ as the output and $u(t)$ as the input.

2. (15%) The mechanical system shown in Figure 2 (a) is used as part of the unity feedback system shown in Figure 2 (b). Find the values of M and D to yield 20% overshoot and 2 seconds settling time.

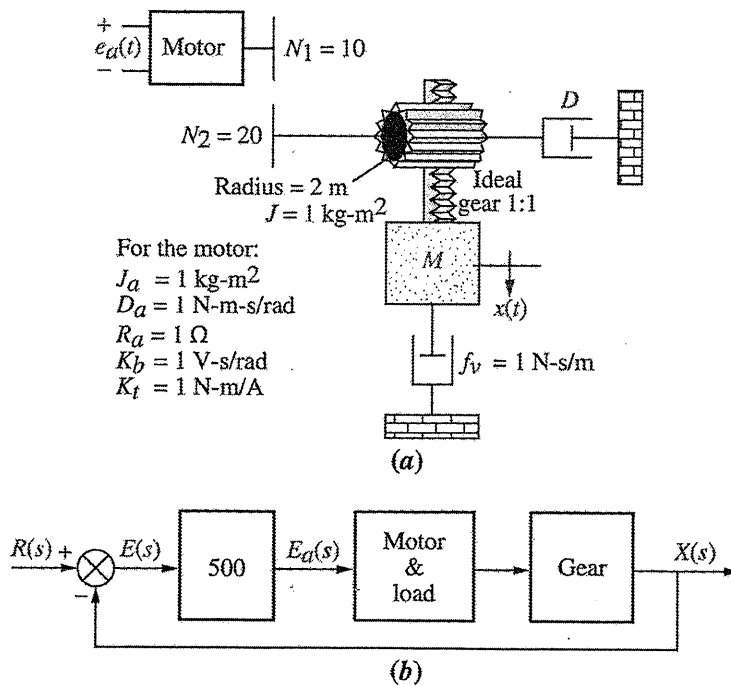


Figure 2. Motor and load in a feedback control system

3. (30%) An example of an automated highway system is shown in Figure 3. A velocity control system for maintaining the *velocity* between vehicles is illustrated in Figure 4. The output $Y(s)$ is the relative velocity of the two automobiles; the input $R(s)$ is the desired relative velocity between the two vehicles. Our design goal is to develop a controller that can maintain the prescribed velocity between the vehicles and maneuver the active vehicle as commanded.

The **CONTROL GOAL** of the system is to maintain the prescribed velocity between the two vehicles, and maneuver the active vehicle as commanded. Design a controller with your choice from a PD or PI controller to satisfy the following **SPECIFICATIONS**.

- (1) Zero steady-state error to a step input
- (2) Steady-state error due to a ramp input of $e_{ss} < 25\%$ of the input magnitude
- (3) Percent overshoot of $< 5\%$ to a step input
- (4) Settling time of $T_s < 1.5 \text{ sec}$ to a step input (using a 2% criterion to establish settling time)

Hint: you can use the root locus technique for second-order approximation of transient response.

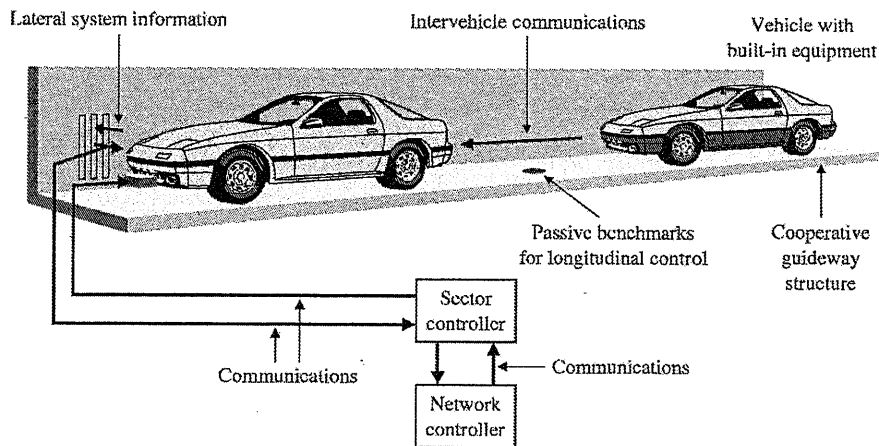


Figure 3. Automated highway system.

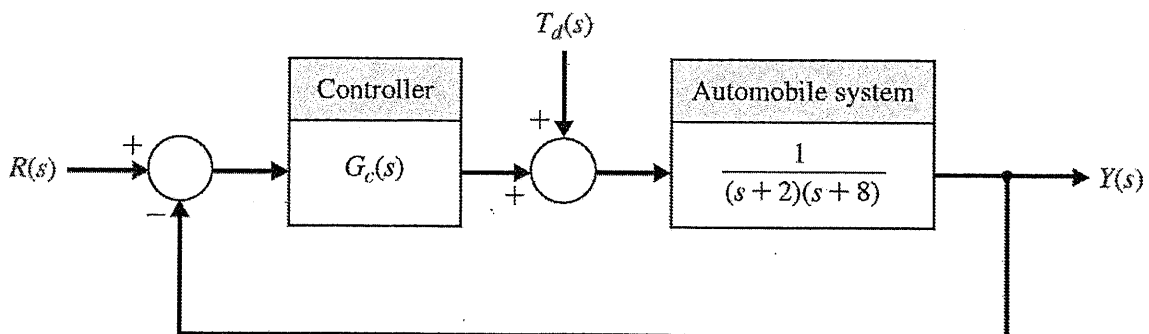


Figure 4. Vehicle velocity control system.

4. (20%) A ship's roll can be stabilized with a control system. A voltage applied to the fins' actuators creates a roll torque that is applied to the ship. The ship, in response to the roll torque, yields a roll angle. Assuming that block diagram for the roll control system shown in Figure 5, answer the followings
- (1) Find out the range of gain for stability, and the oscillation frequency by using the root locus technique.
 - (2) If $K=5$, determine the gain and phase margins for the system via plotting the Nyquist diagram.

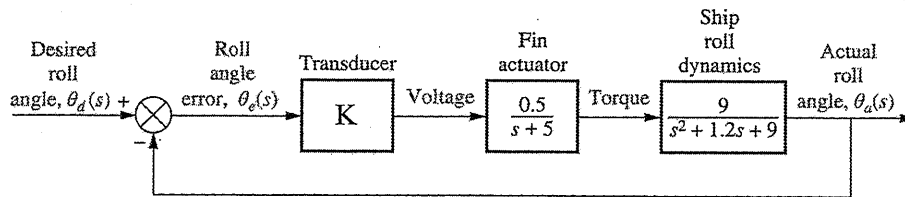


Figure 5. Block diagram of a ship's roll-stabilizing system.

5. (15%) Consider a unity feedback control system with the transfer function

$$\text{System 1: } G(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)} \quad \text{System 2: } G(s) = \frac{50(s+3)}{s(s+2)(s+4)}$$

- (1) Find the analytical expressions for the magnitude and phase response.
- (2) Plot the Bode plot and answer the gain and phase margins.