1. (15%) The delta function is defined as follows,
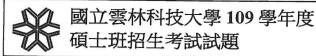
$$\delta(t - a) = \begin{cases} \infty & \text{for } t = a \\ 0 & \text{otherwise} \end{cases}$$

    (a)(5%) Find the Laplace transform $\mathcal{L}\{\delta(t - a)\} = e^{-as}$

    (b)(10%) There is a differential equation $y'' + 2y' + 10y = \delta(t)$, $y(0) = 0$

    and $y'(0) = 1$. Solve the differential equation using Laplace transform.

2. (10%) Solve the differential equation $2xy^2 + 4 = 2(3 - x^2 y)y'$, $y(-1) = 8$.

3. (10%) Solve the differential equation $y'' + 2y' + y = 4e^{-x}$.

4. (15%) Solve the differential equation system $\begin{cases} y_1' = -2y_1 + 5y_2 \\ y_2' = y_1 + 2y_2 \end{cases}$ and sketch its

phase portraits on the phase plane.

5. (15%)Perform the indicated operation, give that:

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 0 & -1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 0 & 1 \end{bmatrix}$$

    (a) A^T (b) (A+B)C   (c)If $2X-4(2A-B)=0$, Find X

6. (10%)IF $\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, Please find $\begin{bmatrix} x \\ y \end{bmatrix}$

7. (10%)Solve the system using either Gaussian elimination with back-substitution.

$$6x - 2y + z = 29$$
$$0x + 4y - 2z = 2$$
$$4x + 8y - 4z = 24$$

8. (15%)Find eigenvalues and eigenvectors of A square matrix.

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$