

科目：微積分

適用：財金系

編號：264

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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Hint: 每空格 5 分，共 100 分。不需列出計算過程

1. Answer following questions.

(a) Find the sum of the geometric series $\sum_{n=0}^{\infty} \left(\frac{2^n}{3^{n+1}}\right)$ if it is convergent.

Ans: _____.

(b) When the fourth Taylor polynomial for $f(x) = \frac{x^2}{1+x^2}$ at $x=0$ is used to approximate the integral $\int_0^1 \frac{x^2}{1+x^2} dx$, the value that is obtained is _____.

(c) Evaluate: $\lim_{n \rightarrow \infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) =$ _____.

(d) $\lim_{x \rightarrow a} [2f(x) - 3g(x)] =$ _____, given that $\lim_{x \rightarrow a} f(x) = 3$ and $\lim_{x \rightarrow a} g(x) = 4$.

2. Evaluate the following integral.

(a) Find the indefinite integral $\int \frac{e^{-2/x}}{x^2} dx =$ _____.

(b) Find the indefinite integral $\int_e^{e^2} \frac{(\ln x)^4}{x} dx =$ _____.

(c) Evaluate the definite integral $\int_0^1 (2x+1)\sqrt{x^2+x} dx =$ _____.

(d) Find the area of the region bounded by the graphs of the functions $f(x) = -x^2 + 4x$ and $g(x) = 2x - 8$. Ans: _____.

(e) Evaluate $\int_0^1 \int_{x^2}^x (3x^3 y^2) dy dx =$ _____.

3. Find the derivatives (or partial derivatives) of the following function.

(a) Find the derivative of the function $f(x) = (x^2 + 1)\left(2x^2 - \frac{1}{x}\right)$.

Ans: _____.

(b) Find the derivative of the function $f(x) = \ln(e^{x^2} + 4x - 10)$.

Ans: _____.

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(c) Let $H(x) = \frac{f(x)}{g(x)}$, with f and g differentiable at $x = 2$. Find

$$H'(2) = \underline{\hspace{2cm}} \quad \text{if } f(2) = 4, f'(2) = 3, g(2) = -2, \text{ and } g'(2) = 5.$$

(d) Let $f(x, y, z) = e^{xz} + xyz^2$. Find $\frac{\partial^2 f}{\partial z^2} = \underline{\hspace{2cm}}$.

(e) Let $f(x, y, z) = 3x^2yz + 2xy^2z^3$. Find $f_{zy} = \underline{\hspace{2cm}}$.

4. A company has a revenue function of $R(x) = 3x^2 + 4y^2 + 6x + 12y + 300$ and a cost function of $C(x) = 4x^2 + 5y^2 + 100$, where R and C are measured in dollars and x and y represent the number of units of two types of products which are produced and sold.

- Find the profit function $P(x) = \underline{\hspace{2cm}}$.
- Find the values of x and y which result in a maximum profit.
Ans: $\underline{\hspace{2cm}}$.
- What is the maximum profit? Ans: $\underline{\hspace{2cm}}$.

5. Let $f(x) = \sqrt[3]{x+1}$.

- Find the interval(s) where $f(x)$ is concave upward. Ans: $\underline{\hspace{2cm}}$.
- Find the interval(s) where $f(x)$ is concave downward. Ans: $\underline{\hspace{2cm}}$.
- Find the x -coordinate(s) of any point(s) of inflection. Ans: $\underline{\hspace{2cm}}$.

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