國立中山大學 109 學年度
碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班電波聯合碩士班選考、通訊所碩士班甲組、乙組選考、電機系碩士班戊組選考】

—作答注意事項—

考試時間：100 分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，不得另攜帶紙張，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，其後果由考生自行負擔。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

1. (10%) Explain what carrier and symbol synchronization are in a communication system.

2. (10%) Consider a random variable X with the distribution:

$$p(X=k)=p(1-p)^{k-1},\ k=1,2,3,4,\ldots.$$

Show that $H(X)=-\log_2 p-\dfrac{1-p}{p}\log_2(1-p)$.

Hint: $H(X)=E[-\log_2 p(X)]$.

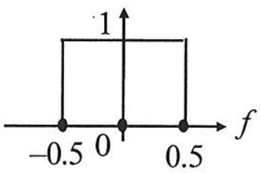
3. (25%) Consider a passband signal given by

$$x(t)=\text{sinc}^2(t)\left(1+A\cos 4\pi t\right)\cos\left(w_c t+\frac{\pi}{6}\right),$$

where $w_c \gg 4\pi$.

(a). (10%) Plot the frequency response $|X(w)|$ where $X(w)$ is the Fourier transform of $x(t)$.

Hint: $\text{sinc}(t) \xrightarrow{\mathcal{F}}$



(b). (5%) Find the equivalent baseband signal of $x(t)$.

Hint: $\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$.

(c). (10%) Find the Hilbert transform of $x(t)$.

Hint: Let Hilbert transform of $x(t)$ be $\hat{x}(t)$. $\mathcal{F}\{\hat{x}(t)\}=-j\,\text{sgn}(w)\,\mathcal{F}\{x(t)\}$.

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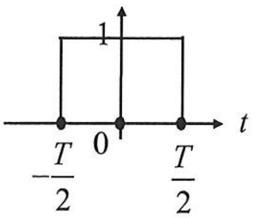
4. (20%) Consider a sequence $\{a_n\}_{n=-\infty}^{\infty}$ of independent and identically distributed (i.i.d.) random variables. Each a_n takes value of +1 and -1 with equal probability. Let the transmit sequence $b_n = a_n - a_{n-1}$. The sequence is then transmitted by the baseband signal $s(t)$, which is given by

$$s(t) = \sum_{n=-\infty}^{\infty} b_n g(t - nT),$$

where $g(t)$ is shown in Fig. 1.

(a). (10%) Decide the Fourier transform of $g(t)$.

Hint: 

$\xrightarrow{\mathcal{F}} \dfrac{2\sin\frac{T}{2}w}{w}$

(b). (10%) Decide the power spectrum density of $s(t)$.

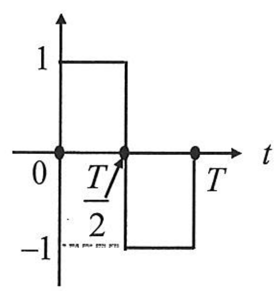


Fig. 1

5. (20%) Consider a real signal $x(t)$ with its Fourier transform $X(jw)$ as shown in Fig. 2. We want to reconstruct the signal $x(t)$ from $x_p(t)$ of the sampling system provided in Fig. 3, where $w_c = \frac{1}{2}(w_1 + w_2)$, and the cutoff frequency of the low-pass filter $H(jw)$ is $\frac{1}{2}(w_2 - w_1)$.

(a). (5%) Plot the spectrum of $x_p(t)$.

(b). (5%) Decide the maximum sampling period T so that $x(t)$ can be perfectly reconstructed from $x_p(t)$.

(c). (10%) Plot a system that can reconstruct $x(t)$ from $x_p(t)$.

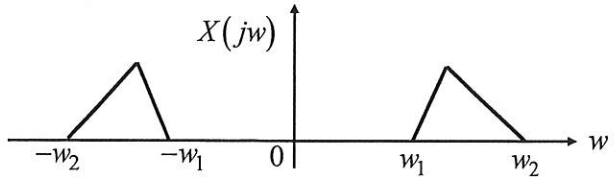


Fig. 2

試題請隨卷繳回，請留意背面是否有題

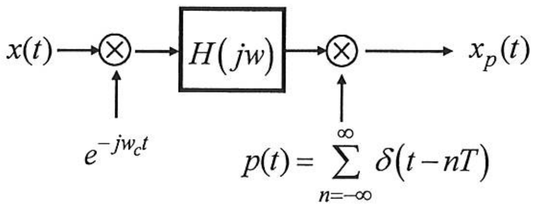


Fig. 3

6. (15%) Consider a communication system as follows

$$Y_i = A_i X_i + N_i,\ i = 1,2,3,4,$$

where $\{X_i\}_{i=1}^4$ are i.i.d. $\mathcal{N}(0,1)$, and $\{N_i\}_{i=1}^4$ are i.i.d. $\mathcal{N}(0,\sigma^2)$. $\mathcal{N}(m,\sigma^2)$ indicates the Gaussian distribution with mean m and variance σ^2. $\{Y_i\}$ are the observed output signals. Let U be the input single binary random variable. If $U=0$, then $A_1 = A_2 = 1$, and $A_3 = A_4 = 0$; $U=1$, then $A_1 = A_2 = 0$, and $A_3 = A_4 = 1$. The input random variable U is independent of $\{N_i\}_{i=1}^4$ and $\{X_i\}_{i=1}^4$.

(a). (5%) Define $\mathbf{Y} = [Y_1\ \ Y_2\ \ Y_3\ \ Y_4]^T$. Determine the joint probability density function $f_{\mathbf{Y}/U}(\mathbf{y} \mid U=0)$.

(b). (5%) Decide the log likelihood ratio

$$LLR(\mathbf{y}) = \ln\left(\frac{f_{\mathbf{Y}/U}(\mathbf{y} \mid U=0)}{f_{\mathbf{Y}/U}(\mathbf{y} \mid U=1)}\right).$$

(c). (5%) Define $E_A = Y_1^2 + Y_2^2$ and $E_B = Y_3^2 + Y_4^2$. Can the difference of $E_A - E_B$ be the sufficient statistics for $LLR(\mathbf{y})$? Provide your justification.