

# 國立臺北大學 109 學年度碩士班一般入學考試試題

系(所)組別：通訊工程學系

科目：機率

第1頁 共1頁

可 不可使用計算機

- (5% each, 20% in total) Yes/No questions. No need to state reasons.
  - Consider three events  $E_1$ ,  $E_2$ , and  $E_3$ . Is the statement “if  $P[E_1 \cap E_2 \cap E_3] = P[E_1]P[E_2]P[E_3]$ , then  $E_1$ ,  $E_2$ , and  $E_3$  are independent” true?
  - Consider two events  $E_1$  and  $E_2$ . Is the statement “if  $E_1 \cap E_2 = \emptyset$ , then  $P[E_1 \cup E_2] = P[E_1] + P[E_2]$ ” true?
  - Let  $\Omega$  be a sample space. Let  $E_1$  and  $E_2$  be two disjoint events, with  $E_1 \cup E_2 = \Omega$ . Is the statement “ $P[E] = P[E \cap E_1] + P[E \cap E_2]$  for any event  $E \subseteq \Omega$ ” true?
  - Consider a random variable  $X$ . Let random variable  $Y = a \cdot X + b$ , for some constants  $a$  and  $b$ . Let  $Var[X]$  and  $Var[Y]$  be the variances of  $X$  and  $Y$ , respectively. Is the statement “ $Var[Y] = a \cdot Var[X] + b$ ” true?

- (5% each, 15% in total) Consider a random variable  $X$  with the PMF

$$P_X(x) = \begin{cases} c & \text{if } x = 1; \\ \left(\frac{1}{2}\right)^x & \text{if } x = 2, 3, 4, \dots; \\ 0 & \text{else,} \end{cases}$$

for some constant  $c$ .

- What is the constant  $c$ ?
  - What is the probability  $P[X \in \{0, 1\}]$ ?
  - What is the mean  $E[X]$ ?
- (5% each, 15% in total) Let  $X$  be the number of accesses at a website in a day. Suppose that  $X$  is the Poisson random variable with mean  $\alpha$ .
    - What is the probability that no one accesses the website in that day?
    - What is the probability that no one accesses the website during 10am and 11am in that day?
    - Suppose that each access at the website consumes a power of  $c$  units, i.e.,  $c \cdot X$  units of power consumption in that day. What is the expected power consumption in that day?
  - (2% for (a), and 4% for (b)-(e), totally 18%) Let  $X$  be a random variable with the following probability density function (p.d.f.)

$$f_X(x) = \begin{cases} cx, & 0 \leq x \leq 2; \\ 2c(3-x), & 2 < x \leq 3; \\ 0, & \text{otherwise.} \end{cases}$$

where  $c$  is a constant.

- Find the value of  $c$ .
  - Find the mean of  $X$ .
  - Find the variance of  $X$ .
  - Find the median of  $X$ .
  - Let  $Y = X^2$ . Find the mean of  $Y$ .
- (6% for each, totally 24%) Let  $X$  and  $Y$  be two independent random variables with identical uniform distribution over  $[0, 2]$ . Let  $Z = |Y - X|$ .
    - Find the probability  $P(X < 2Y)$ .
    - Find the conditional probability  $P(2X < Y | X < 2Y)$ .
    - Find the cumulative distribution function (c.d.f.) of  $Z$ .
    - Find the p.d.f. of  $Z$ .
  - (8%) Let  $X$  and  $Y$  be two random variables.  $X$  has the p.d.f.  $f_X(x) = 2x, 0 \leq x \leq 1$ , and  $Y = g(X)$ . If  $Y$  has the p.d.f.  $f_Y(y) = e^{-y}, y > 0$ , find the function  $g(x)$ .

試題隨卷繳交