1. (10%) Consider an RSA cryptosystem with $p = 11$, $q = 29$, and public-key $(e, n) = (3, 319)$. What is the value of d used in the secret-key? What is the encryption of the message $M = 100$?

2. (20%) Consider the graph below. Answer the questions and prove your answer. Does it have an Euler circuit? Does it have an Euler path? Does it have a Hamilton circuit? Does it have a Hamilton path?
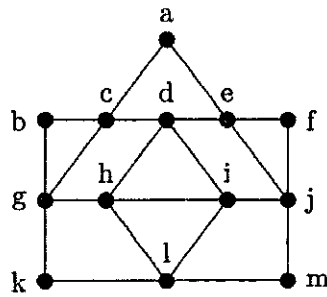
3. (20%) Answer the following questions.

    (a) In how many ways are there to arrange the letters of the word "APPLE"?

    (b) How many of these ways start or end with the letter P?

    (c) Find the number of solutions to $x + y + z = 25$, where x, y, and z are nonnegative integers.

    (d) Answer part (c), but assume that $x \geq 3$ and $y \geq 8$.

4. Give the matrix $\mathbf{A} = \begin{bmatrix} -6 & -6 & 6 \\ 3 & -3 & -3 \\ 2 & 6 & 2 \end{bmatrix}$.

    (a) (10%) Compute the LU decomposition of $\mathbf{A}$.

    (b) (5%) Determine the nullspace of $\mathbf{A}$.

    (c) (5%) Give a basis for the column space of $\mathbf{A}$.

5. Give the matrix $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$.

    (a) (10%) Find the eigenvalues and the eigenvectors of $\mathbf{B}^{\mathrm{T}}\mathbf{B}$.

    (b) (10%) Find the singular value decomposition of $\mathbf{B}$.

6. (10%) Assume that both $\mathbf{C}$ and $\mathbf{D}$ are $n \times n$ matrices. Determine whether $L(\mathbf{D}) = \mathbf{C}\mathbf{D}^3$ is a linear operator from $\mathbf{R}^{n \times n}$ to $\mathbf{R}^{n \times n}$.

試題隨卷繳交