

# 國立臺北大學 109 學年度碩士班一般入學考試試題

系(所)組別：統計學系  
科目：數理統計

第1頁 共1頁

可 不可使用計算機

1. (50%) Let  $X_1, X_2, \dots, X_n$  be a random sample of size  $n$  from a distribution that has pdf

$$f(x) = \frac{1}{\theta} \exp\left[-\left(\frac{x-\alpha}{\theta}\right)\right], \alpha \leq x < \infty.$$

- (a) (5%) What is the mean of  $X_1$ ? Justify your answer.
- (b) (10%) DERIVE the cumulative distribution function of  $Y_n$ , which is defined as the minimum of  $X_1, X_2, \dots, X_n$ . (Do not use the formula for the CDF of order statistics.)
- (c) (10%) Does  $Y_n$  converge in probability? If yes, what random variable does it converge to? Prove the convergence or divergence.
- (d) (10%) Does  $Y_n$  converge in distribution? If yes, what distribution does it converge to? Prove the convergence or divergence.
- (e) (5%) Why do you think statisticians are interested in the convergence of random variables?
- (f) (10%) How does (a) to (e) give you any thought on the estimation for  $\theta$ ? Briefly describe your thoughts, and justify your thoughts.

2. (50%) Let  $X_1, X_2, \dots, X_n$  be a random sample from a normal distribution

$$f(x; \theta) = (2\pi\theta)^{-1/2} \exp\left(-\frac{1}{2\theta}(x-\mu)^2\right), x \in R; \mu \in R, \theta > 0.$$

Let the distribution function of  $f(x; \theta)$  be denoted as  $\Phi(x; \mu, \theta)$ .

- (a) (10%) Find the maximum likelihood estimators  $\hat{\mu}, \hat{\theta}$  of  $\mu, \theta$ .
- (b) (10%) Find the Rao-Cramér lower bound of  $\hat{\mu}$  and  $\hat{\theta}$ , respectively.
- (c) (10%) Find the asymptotic distribution of  $\hat{\mu}, \hat{\theta}$ .
- (d) (10%) Find the asymptotic distribution of  $\Phi(x; \hat{\mu}, \hat{\theta})$ . [Hint: Use Delta method and the derivatives with respect to  $\mu$  and  $\theta$  do not need to be expanded.]
- (e) (10%) Find the Likelihood ratio test (LR test) for testing  $H_0: \theta = \theta_0$  versus  $H_0: \theta \neq \theta_0$ , where  $\theta_0$  is a pre-specified value and provide the corresponding critical value for the LR test.

試題隨卷繳交