國立臺北大學 109 學年度碩士班一般入學考試試題

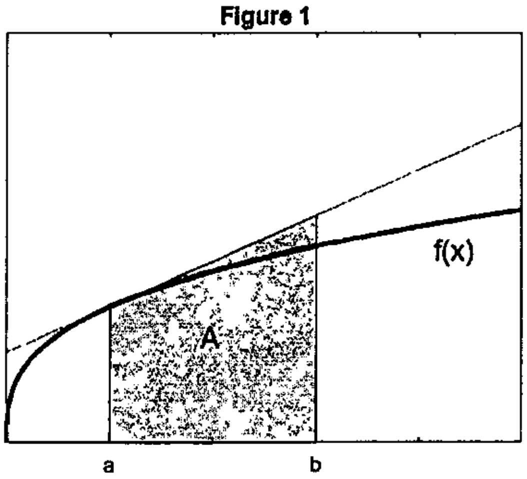
系（所）組別： 統計學系
科　　　目： 基礎數學

□可 ☑不可使用計算機

一、(50%) CALCULUS

1. (10%) Let $f(x)$ be a differentiable function as shown in Figure 1. It needs to compute the integral $\int_a^b f(x)dx$. For some reason, the integration is not possible. We decide to linearize $f(x)$ at $x = a$ and use the trapezoidal area A to approximate the integral, i.e. $\int_a^b f(x)dx \approx A$. Please find the area A.

Figure 1



2. (10%) Sketch the graph of a function $f(x)$ which has all of the following properties:

   (a) $\lim_{x\to 1^+} f(x) = \infty, \lim_{x\to 1^-} f(x) = -\infty, \lim_{x\to -\infty} f(x) = 0$

   (b) $f(-2) = 3,\ f(5) = 1,\ f(0) = 0$

   (c) $f'(5) = 0,\ f'(-2) = 0$

   (d) $f'(x) > 0$ if $x < -2$ or $x > 5$, $f'(x) < 0$ if $-2 < x < 1$ or $1 < x < 5$

   (e) $f''(x) > 0$ if $x < -3$ or $x > 1$, $f''(x) < 0$ if $-3 < x < 1$

3. (10%) Evaluate the integral

$$E = \int_0^L \frac{\lambda c}{4\pi\delta(x^2 + c^2)^{3/2}} dx$$

Where c, λ, δ, L are constants.

4. (10%) Evaluate the double integral

$$\int_0^2 \int_{x^2}^4 x e^{-y^2} dy dx$$

試題隨卷繳交　　　　　　接背面

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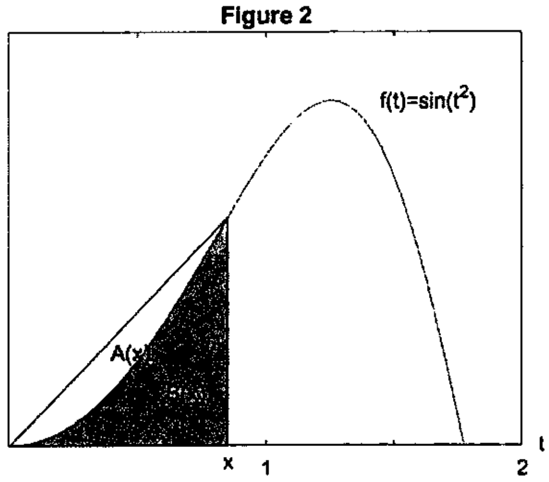
5. (10%) Let the function $f(t) = sin(t^2)$ as shown in Figure 2. Consider two areas $A(x)$ and $B(x)$ for $x < 1.5$, where $A(x)$ represents the triangular area, while $B(x)$ designates the definite integral of $f(t)$ from $t = 0$ to $t = x$. Find

$$\lim_{x \to 0^+} \frac{A(x)}{B(x)}$$

Figure 2



二、(50%)（所有題目請敘述計算過程，無計算過程不給分）

1. Let $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

   (a) Write down the characteristic polynomial of A and use it to find the eigenvalues. (4%)

   (b) Find the eigenspaces of A. (5%)

   (c) Orthogonally diagonalize the matrix A. (You need to find out an orthogonal matrix P and a diagonal matrix D such that $P^T AP = D$.) (10%)

2. Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$ be vectors in a vector space V. Please show that

   (a) $\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ is a subspace of V. (10%)

   (b) $\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ is the smallest subspace of V that contains $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$. (4%)

3. Let $S = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ be an orthogonal set of nonzero vectors in a vector space V. Please show that the vectors in S are linear independent. (10%)

4. Let $T : P_2 \to P_2$ be the linear transformation defined by

$$T(p(x)) = p(2x - 1).$$

Please find the matrix of T with respect to the basis $\mathcal{E} = \{1, x + 1, x^2\}$. (7%)

試題隨卷繳交