

國立臺北大學 109 學年度碩士班一般入學考試試題

系(所)組別：都市計劃研究所甲組

科目：微積分

第1頁 共1頁

可 不可使用計算機

1. (10%) Consider a function

$$f(x) = \begin{cases} 1.5x^2 - 0.5x + 1 & \text{if } x \leq 0 \text{ and } x \neq -1 \\ \frac{x+1}{2} & \text{if } x \leq -1 \\ -\frac{1}{24}x^3 + \frac{3}{4}x^2 - \frac{71}{24}x + \frac{21}{4} & \text{if } 0 < x \leq 1 \\ & \text{if } x > 1 \text{ and } x \neq 5 \end{cases}$$

A. Find the following limit.

i. $\lim_{x \rightarrow -1} f(x)$

ii. $\lim_{x \rightarrow 0} f(x)$

iii. $\lim_{x \rightarrow 1} f(x)$

iv. $\lim_{x \rightarrow 5} f(x)$

B. Find the following derivatives.

i. $f'(0)$

ii. $f'(5)$

iii. $f'(1)$

iv. $f'(-1)$

2. (10%) Suppose the function $f(x) = x^3 + ax^2 + bx - 3$ has extreme values at $x = 1, -3$. Find a and b .

3. (20%) Evaluate the following limit.

A. $\lim_{x \rightarrow \infty} \left(\frac{2x^3 - x}{x^2 + x - 2} - 2x \right)$

B. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x)$

4. (20%) Evaluate the following integrals.

A. $\int_0^5 x\sqrt{9-x} dx$

B. $\int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2+3}} dx$

C. $\int \frac{2e^x}{2+e^x} dx$

D. $\int \frac{1}{6-5x+x^2} dx$

5. (10%) Find the local extrema of $f(x) = 3x^5 - 5x^3$, where $-2 \leq x \leq 2$.

6. (10%) Find the tangent line of $x^4 - 3x^2y + y^4 = -1$ passing through $(1,1)$.

7. (10%) Find $\iint_R \frac{1}{x \ln y} dA$, where $R = \{(x,y) | 0.1 \leq x \leq 1, 0.1 \leq y \leq x\}$.

8. (10%) Find the volume of the solid generated by rotating the region bounded by $y = \sin x$, $x = 0$, $x = \pi$ and x axis.