國立臺北大學 109 學年度碩士班一般入學考試試題

系（所）組別：經濟學系

科　　目：統計學

☑可 ☐不可使用計算機

I. 選擇題（每題 4 分，請在作答時標示大題「I」及其小題號(1)、(2)、.....(5)，不須另外標示 1.、2.、3.等）

1. Jack use OLS method to estimate the parameters of linear model with intercept:

   $y_t = \alpha + \beta x_t + \varepsilon_t$.

   If the heteroskedasticity ($Var(\varepsilon_t \mid x_t) = \sigma_\varepsilon^2 x_t$) exits in the errors but other Gauss-Markov conditions satisfy, which statement is true? ____(1)____

   (A). The estimated parameter, $\hat{\beta}$, is biased, and variance of $\hat{\beta}_1$ obtained by OLS method is incorrect.

   (B). The estimated parameter, $\hat{\beta}$, is biased, and variance of $\hat{\beta}_1$ obtained by OLS method is accurate.

   (C). The estimated parameter, $\hat{\beta}$, is unbiased, and variance of $\hat{\beta}_1$ obtained by White heteroskedasticity-robust method is accurate.

   (D). The estimated parameter, $\hat{\beta}_1$, is unbiased and efficiency.

   (E). Jack cannot use OLS to estimate parameters because of inefficiency.

2. Consider the simple linear regression model:

   $y_t = \beta y_{t-1} + \varepsilon_t$,

   with the error structure as below:

   $\varepsilon_t = \rho \varepsilon_{t-1} + u_t$,

   where $u_t \sim i.i.d(0, \sigma^2)$. What is the probability limit ($p\lim$) of OLS estimator $\hat{\beta}$? ____(2)____

   (A). $\dfrac{\beta}{1+\beta\rho}$

   (B). β

   (C). $\dfrac{\beta+\rho}{1+\beta\rho}$

   (D). $\dfrac{\rho}{1+\beta\rho}$

   (E). $\dfrac{1}{1+\beta\rho}$

3. Consider the model:

   $y_t = \alpha + \rho x_t + \varepsilon_t$,

   where $t = 1, \ldots T$. If we want to obtain consistent estimator $\hat{\rho}$ for the large sample, which necessary condition it must be have?

   ____(3)____

   (A). $\text{cov}(\varepsilon_t, x_t) = 0$

   (B). $Var(\varepsilon_t \mid 1, x_t) = \sigma^2$

   (C). $E(\varepsilon_t \mid 1, x_t) = 0$

   (D). $E(\varepsilon_t \mid x_t) = 0$

   (E). $E(\varepsilon_t \varepsilon_s \mid x_t, x_s) = 0$

4. An economist uses data to estimate the linear model with intercept

$y_t = \alpha + \beta_1 x_{1t} + \varepsilon_t$,

but the true population model is as below:

$y_t = \alpha + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t$.

Under which condition satisfied, the OLS estimator $\hat{\beta}_1$ is still consistent? ____(4)____

(A). $E(x_{2t}u_t) = 0$

(B). $\alpha = 0$

(C). $E(x_{2t}\varepsilon_t) = 0$

(D). $E(x_{2t} \mid x_{1t}) = 0$

(E). $E(x_{1t}u_t) = 0$

5. Suppose that there is an exact relationship between two scalar random variable y and x:

$y = \beta x + v$

However, y is not known and unobserved. A variable x^* is observed, such that

$x^* = x + u$

Under which condition satisfied, the OLS stimator $\hat{\beta}_1$ is still consistent? ____(5)____

(A). $E(yx^*) = 0$

(B). $E(x^*u) = 0$

(C). $E(xu) = 0$

(D). $E(xv) = 0$

(E). $E(uv) = 0$

II. 計算問答題（配分如標示，請在作答時標示大題「Ⅱ」及其小題號(6)、(7)、……(9)，不須另外標示 1-1、1-2 等）

1. You have data $(x_t \quad y_t)$, t = 1, ..., T. Suppose that the model

$y_t = \alpha + \beta x_t + \epsilon_t$

satisfies Gauss-Markov conditions Let x_B be the biggest x, x_S be the smallest x, and y_B, y_S are the values corresponding to x_B, x_S. (Note: y_B may or may not be the largest y; it is the y corresponding to the largest x. Similarly for y_S.)

1-1 (5 分) Define

$$\tilde{\beta} = \frac{y_B - y_S}{x_B - x_S}$$

Whether $\tilde{\beta}$ is an unbiased estimator? ____(6)____

1-2 (10 分) Please calculate $\text{var}(\tilde{\beta})$. Whether $\tilde{\beta}$ is efficiency? ____(7)____

2. Robert has T = 100 observations on variables y, x_1, x_2 and x_3. He estimates the following equations by least squares:

Equation (1): $y = -4.0 + 2.0\,x_1 - 3.0x_2 + 0.04x_3$

(2.4) (-0.5) (?)

Equation sum of squares (SSE)=50, SST=100, Durbin-Watson "d" =1.1

where the numbers in parentheses are t-statistic.

2.1 (10 分) What is the true value of R^2 in Equation (1)? What is the value of s^2 (estimated error variance.)? ____(8)____

2.2 (5 分) What is the Durbin-Watson statistic in Equation (1)? ____(9)____

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III.計算問答題（配分如標示，請在作答時標示大題「Ⅲ」及其小題號(10)、(11)、.....(16)，不須另外標示 1.、2.、3.等）

1. (8 分)假設隨機抽樣所來自的母體可以區分成兩類人群，A 群與 B 群。A 群人所構成的子母體佔總母體比例為 60% 且其子母體的變異數σ_A為 7；B 群人所構成的子母體佔總母體比例為 40% 且其子母體的變異數σ_B為 5。考慮隨機從總母體抽出 N 個人（不分 A、B 群），並計算其樣本平均數$\overline{Y}$。若 A、B 兩群子母體有相同的母體平均，即$\mu_A = \mu_B = \mu$，則$\sqrt{N}(\overline{Y} - \mu)$的漸近分配為何。 ＿＿(10)＿＿

2. 阿正手上展示一個有正反兩面不同花樣的銅板（正面以 H 表示，反面以 T 表示），令p代表隨機投擲一次會出現正面的機率。小飛與小貝欲估計p的值，並要求阿正給點提示。阿正於是擲投此銅板三次得到三個正面，以 HHH 表示。

   2.1 小飛是個頻率學派統計學家(frequentist statistician)，且打算使用最大概似估計法(Maximum Likelihood Estimation)來估計p。

   (7 分)請寫下小飛的樣本概似函數，並求解其最大概似估計值（簡稱 MLE 值）。＿＿(11)＿＿（過程要清楚寫下一階條件，如有角解也需清楚呈現）

   2.2 小貝是貝氏學派統計學家（Bayesian statistician），她在還未看到阿正的樣本前其對p認知的先驗機率分配（prior distribution）如下表所示：

p	機率
0.25	0.25
0.50	0.25
0.75	0.25
1.00	0.25

   (8 分)在看到阿正的樣本後，小貝對p認知的事後機率分配為何？ ＿＿(12)＿＿

   (7 分)若小貝打算用$\mathbb{E}(p|HHH)$來推估p，則她的估計值為多少？＿＿(13)＿＿

   (6 分)比較小飛與小貝的估計，誰的值比較合理？理由為何？＿＿(14)＿＿（**此題得分以你的統計邏輯依狀況給分，沒有一定對錯，請盡量陳述發揮。**）

3. 假設一台機器平均一天故障 0.05 次，若故障次數服從 Poisson 分配。

   (7 分)請問一個月（以 30 天計）故障不超過 3 次的機率為何？＿＿(15)＿＿

   （**註：$\exp(-0.05) = 0.9512, \exp(-0.1) = 0.9048, \exp(-0.2) = 0.8187, \exp(-0.5) = 0.6065$**）

   (7 分)二次故障間隔超過 30 天的機率為何？＿＿(16)＿＿

試題隨卷繳交