國立臺灣科技大學 109 學年度碩士班招生試題

系所組別：資訊工程系碩士班

科　　目：計算機數學

(總分為 100 分)

1. Given $E = Span((1,2,1)^T, (-2,-3,2)^T, (10,17,-2)^T)$,

   (a) (9%) Find an orthonormal basis of $E$.

   (b) (5%) Find its orthogonal complement $E^{\perp}$.

2. Suppose that there are only three companies A, B, and C in Taiwan's mobile telecom market. Each year 20% of A's customers discontinue their packages and jump to B, 20% jump to C, and the other 60% remain. For company B, 10% of customers jump to A, 20% jump to C, and the others remain. For company C, 20% of customers jump to A, 10% jump to B, and the remaining customers stay in C.

   (a) (12%) Write down the transition matrix $A$ and calculate its eigenvalues $\lambda$.

   (b) (6%) Assume that the initial market share vector of A, B, and C is $\begin{bmatrix}1\\0\\0\end{bmatrix}$, what is the limit $A^k\begin{bmatrix}1\\0\\0\end{bmatrix}$ as $k \to \infty$?

3. Let $B = \{3+2x+x^2, -x+2x^2, 1+x^2\}$, $T : \mathbf{P}_3 \mapsto \mathbf{P}_3$ be the transformation $T(p(x)) = 2p(x) - p'(x)$.

   (a) (6%) Use 2 different approaches to show that $B$ is a basis for $\mathbf{P}_3$, the vector space of all polynomials of degree less than 3.

   (b) (4%) Calculate $[1]_B$.

   (c) (8%) Write down the matrix T with respect to the basis $B$.

4. (15%) Solve the system of congruences $x \equiv 5 \pmod 6, x \equiv 3 \pmod{10}, x \equiv 8 \pmod{15}$, and $x \equiv 11 \pmod{21}$.

5. (10%) Consider the seating arrangements of five men $m_1, m_2, m_3, m_4$, and $m_5$, and five women $w_1, w_2, w_3, w_4$,and $w_5$ at a round table, where two seating arrangements are considered the same if one of them can be derived from the other by a rotation. What is the probability that $m_1$ and $w_1$ sit next to each other, while there are exactly two people sitting between $m_2$ and $w_2$?

6. (10%) Prove that if $G$ is an undirected graph with five vertices and every vertex of $G$ has degree 2, then $G$ must be a cycle.

7. (15%) A space probe communicates with Earth using bit strings. Suppose in its transmissions the probe sends a 1 one-fourth of the time and a 0 three-fourths of the time. Also, when a 0 is sent to Earth, the probability that a 0 is received is 0.9, and the probability that a 1 is received is 0.1. When a 1 is sent to Earth, the probability that a 1 is received is 0.8, and the probability that a 0 is received is 0.2. Suppose that every bit is processed independently during transmission at either end of the communication, what is the probability that the received bit string "101" is correct?