國立臺灣科技大學 109 學年度碩士班招生試題

系所組別：材料科學與工程系碩士班乙組
科　　目：工程數學

(總分為 100 分)

1. Solve the given differential equation.　(15%)

$$6x^2y + 12xy + y^2 + (6x^2 + 2y)y' = 0$$

2. Solve the given differential equation.　(15%)

$$y''' + 2y'' + 2y' = 2 - 4x^2$$

3. Solve the given integrodifferential equation.　(12%)

$$y' + 4y = 4\int_0^t (\sin\tau)y(t-\tau)d\tau \qquad y(0) = 1$$

4. Find the inverse Laplace transform of the given function.　(8%)
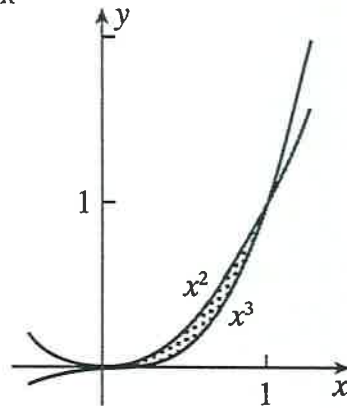
$$F(s) = \frac{s^2 e^{-3s}}{(s-1)^2(s+1)}$$

(總分為 100 分)

5. Please calculate the double integral over the region R that is bounded by the graph of the given equation. (10%)

$$\iint_R (2x+4y+1)dA, \quad y=x^2, \quad y=x^3$$

6. Please find a matrix P that diagonalizes A and the diagonal matrix D such that D=PA^{-1}P. (15%)

$$\mathbf{A} = \begin{pmatrix} -4 & -5 \\ 8 & 10 \end{pmatrix}$$

7. Please solve the heat equation $k\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t}, \quad 0 < x < t, \quad t > 0$ subject to the given conditions. Assume a rod of length L. (10%)

$$u(0,t)=0, \quad u(L,t)=0, \quad u(x,0)=x(L-x)$$

8. $P_1(0, 0, 1)$, $P_2(0, 1, 2)$, $P_3(1, 2, 3)$, please calculate $\overrightarrow{P_1P_2} \times \overrightarrow{P_2P_3}$ (15%)