國立臺灣科技大學 109 學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組

科　　目：工程數學

(總分為 100 分)

1. Please solve the following Ordinary Differential Equations

(i) $-2xy\sin(x^2) + \cos(x^2)y' = 0$　(15%)

(ii) $(x^4 + y^2)dx - (xy)dy = 0$,　$y(2) = 1$　(15%)

2. (10%) Find the work done by a force $\vec{F} = y^2\,\vec{i} + x^2\,\vec{j} + \cos^2 z\,\vec{k}$

moves a particle along a curve $\vec{r} = \cos t\,\vec{i} + \sin t\,\vec{j} + t\,\vec{k}$ from (1, 0, 0) to (1, 0, 4π).

3. (10%) Find the normal vector of the surface, $f = 4x^2 + y^2 + 9z^2$, at the given point P=(2,4,0), and find the directional derivative in the direction $\vec{a} = [-2, -4, 3]$.

4. (20%) Solve the partial differential equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0,\ \ (0 < x < \infty)$$

subject to the conditions

$$u(0,t) = H(t)e^{-t},\ \ u(x,0) = \frac{\partial u(x,0)}{\partial t} = 0,$$

where $H(t)$ is the Heaviside unit step function.

5. Solve $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3e^t \\ e^t \end{bmatrix} \Leftrightarrow \dot{\vec{r}} = A\vec{r} + \vec{b}$

by diagonalization, and A is diagonalizable.
(1) Find all eigenvalues and eigenvectors. (15%)

(2) Finally find $x(t)$ and $y(t)$. (15%)