國立臺灣科技大學 109 學年度碩士班招生試題
系所組別：自動化及控制研究所碩士班
科　　目：工程數學

(總分為 100 分)

1. (a) Find the solution of the following differential equation

   $y'' - y' + y = 1, y(1) = 4, y'(1) = -2.$ (10%)

   (b) Find the recurrence relation of the following differential equation and generate the first five terms of a power series solution about 0.

   $y'' - (1-x)y' + 2y = 1 - x^2.$ (10%)

2. Use the Laplace transform and convolution to solve the following integral equation $f(t) = -1 + t - 2\int_0^t f(t-\tau)\sin(\tau)d\tau.$ (10%)

3. (a) The period of $f(x)$ is 5. Write the complex Fourier series of $f(x) = e^{-x}$ for $0 \leq x < 5$. Determine what this series converges to. (10%)

   (b) Use convolution to find the inverse Fourier transform of the following function $\frac{1}{(1+i\omega)(2+i\omega)}$. (10%)

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4. Let L: $\mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be the linear transformation defined by $L(\mathbf{x})=(x_2, x_1+x_2, x_1-x_2)^T$. Find the matrix representations of L with respect to the ordered bases $\{\mathbf{u}_1, \mathbf{u}_2\}$ and $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$, where $\mathbf{u}_1=(1, 2)^T$, $\mathbf{u}_2=(3, 1)^T$ and $\mathbf{b}_1=(1, 0, 0)^T$, $\mathbf{b}_2=(1, 1, 0)^T$, $\mathbf{b}_3=(1, 1, 1)^T$. (15%)

5. If $\mathbf{F}(x, y, z)= xz\mathbf{i}+xyz\mathbf{j}-y^2\mathbf{k}$, find the curl and divergence of $\mathbf{F}$. (15%)

6. Let u be harmonic in the interior of a rectangular region $0 \le x \le a$, $0 \le y \le b$, so that

$$u_{xx}(x, y)+u_{yy}(x, y)=0 \qquad (0 < x < a,\ 0 < y < b).$$

These values are prescribed on the boundary (Figure P6):

$$u(0, y)=0,\ u(a, y)=0 \qquad (0 < y < b),$$

$$u(x, 0)=f(x),\ u(x, b)=0 \qquad (0 < x < a).\ (20\%)$$

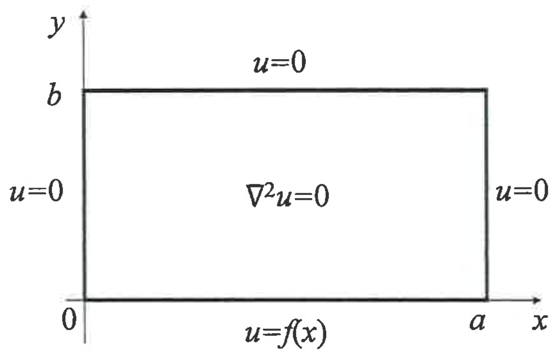

Figure P6