國立臺灣師範大學 109 學年度碩士班招生考試試題

科目：工程數學　　　　　　　　　　適用系所：機電工程學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. （20 分）As shown in Figure 1, an object of mass M is attached to a viscous damper having a viscous coefficient D, and a spring K is connected between the damper and a stationary frame. The mass is placed on a highly varnished table and slides in one dimension in the absence of friction. Let x and f respectively denote the displacement and the force acting on the mass, as shown in Figure 1. Determine the impulse response of the system if the mass is initially at rest. That is, find $x(t)$ subject to $x(0^-) = 0$, $\dot{x}(0^-) = 0$, and $f(t) = \delta(t)$, in which $\delta(t)$ denotes the Dirac delta function.
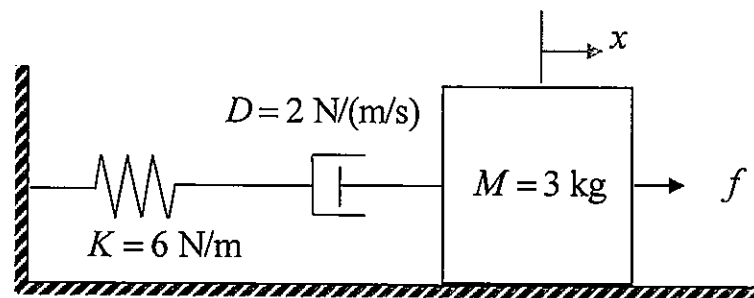

Figure 1. System for Problem 1

2. （20 分）In the circuit of Figure 2, the capacitor is initially discharged, $C = 1\,\mu\text{F}$, $R_1 = 1\,\text{k}\Omega$, and $R_2 = 1\,\text{k}\Omega$. Let V_i and V_o respectively denote input and output voltages of the circuit. Determine the unit-step response of the circuit. That is, find $V_o(t)$ subject to $V_o(0) = 0$, and $V_i(t) = u(t)$, in which $u(t)$ denotes the unit-step function.
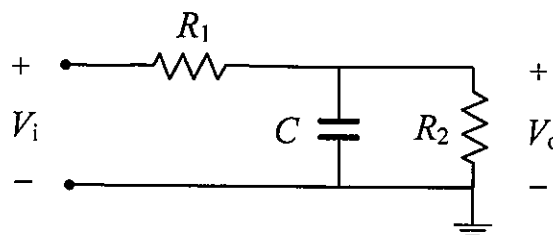

Figure 2. System for Problem 2

3. （15 分）Determine whether or not the following quadratic form is positive definite: $V(x_1, x_2, x_3) = 6x_1^2 + 4x_2^2 + 2x_3^2 + 4x_1x_2 - 4x_2x_3 - 2x_1x_3$.

4. （15 分）Define a function $f(x) = 1 + x$ on the interval $(-1, 1)$. Find the Fourier series of $f(x)$ on the interval $(-1, 1)$.

5. （15 分）A fluid flows with velocity $\mathbf{v} = <z, x^2, y^2>$, where distances are in meters and the components of $\mathbf{v}$ are in meters per second. Find the rate of flow outward through the portion of the cylindrical surface: $x^2 + y^2 = 1$, $0 \le z \le 1$, $y \ge 0$.

6. （15 分）Consider the differential equation $\ddot{x}(t) + 20\dot{x}(t) + 100x(t) = 0$ subject to various initial conditions. The initial values, $\dot{x}(0)$ and $x(0)$, may be represented in a plane with $x(0)$ as the horizontal axis and $\dot{x}(0)$ as the vertical axis. Determine the regions in the $x(0)$-versus-$\dot{x}(0)$ plane where there is no sign change in $x(t)$ for $t \ge 0$. Sketch these regions in the $x(0)$-versus-$\dot{x}(0)$ plane.