國立臺灣師範大學 109 學年度碩士班招生考試試題

科目：代數　　適用系所：數學系

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (10 points) Let G be a group of order pq, where p and q are prime numbers. Show that every proper subgroup of G is cyclic.
2. (20 points) Suppose that N is a normal subgroup of G, H a subgroup of G and define

$$NH = \{nh | n \in N, h \in H\}.$$

Show that

(1) NH is a subgroup of G.
(2) if H is a normal subgroup of G, then NH is also a normal subgroup of G.

3. (20 points) Let G be a finite group of order 45. Then

(1) Find the numbers of Sylow 3-subgroup(s) and Sylow 5-subgroup(s) of G.
(2) Show that G is an abelian group.

4. (10 points) Show that if $1-ab$ is an unit in a ring R, then so is $1-ba$.
5. (20 points) Let $\mathbb{Z}[\sqrt{10}]$ be the ring of real numbers of the form $a+b\sqrt{10}$, where $a, b \in \mathbb{Z}$. Show that $\mathbb{Z}[\sqrt{10}]$ is not a UFD (unique factorization domain).
6. (20 points) Show that

(1) the ideal (41) is not a prime ideal in $\mathbb{Z}[i]$.

(2) the ideal (7) is not a prime ideal in $\mathbb{Z}[\sqrt{2}]$.