

國立臺灣師範大學 109 學年度碩士班招生考試試題

科目：高等微積分

適用系所：數學系

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (15 points) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be differentiable and $\alpha > 0$. If $f(0) = 0$ and $|f'(x)| \leq \alpha$ for all x , how large and how small can $f(5)$ be? Justify your argument.
2. (15 points) Approximate $\cos 0.1$ within the accuracy 10^{-4} .
3. (10 points) Test the convergence or divergence of the series $\sum_{k=1}^{\infty} (\sin^k k) / k^2$.
4. (15 points) Let $f: [a, b] \rightarrow \mathbb{R}$ be continuous. If $\int_a^b f(x)^2 dx = 0$, show that $f(x) = 0$ for all $x \in [a, b]$.
5. (15 points) Let $f: [0, 1] \rightarrow \mathbb{R}$ be continuous. Define $g_k(x) = f(x/k)$ where $x \in [0, 1]$. Show that the sequence $\{g_k\}_{k=1}^{\infty}$ converges uniformly on $[0, 1]$.
6. (15 points) Let $X \subset \mathbb{R}^n$ be an open set. Show that X is a union of open balls.
7. Let $B = \{(x, y) \in \mathbb{R}^2 \mid x, y \in [-1, 1]\}$ and $f(x, y) = x^2 + y^2$.
 - (1) (5 points) Show that f attains its maximal and minimal values on the set B .
 - (2) (10 points) Find the maximal and minimal values of f on the set B .